

Solution to Sunco

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What is the objective?

$$\begin{aligned}\text{Maximize profit} &= \text{income} - \text{cost} \\ &= \text{income} - (\text{crude cost} + \text{production cost} + \text{advertising cost}) \\ &= 70(x_{11} + x_{21} + x_{31}) + 60(x_{12} + x_{22} + x_{33}) + 50(x_{13} + x_{23} + x_{33}) \\ &- (45(x_{11} + x_{12} + x_{13}) + 35(x_{21} + x_{22} + x_{23}) + 25(x_{31} + x_{32} + x_{33})) \\ &- 4 \sum_{i=1}^3 \sum_{j=1}^3 x_{ij} \\ &- (a_1 + a_2 + a_3)\end{aligned}$$

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1. Gas produced meets demand
2. at most 5000 barrels of each crude
3. at most 14000 barrels a day
4. octane levels met
5. sulfur levels met

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1)

$$x_{11} + x_{21} + x_{31} = 3000 + 10a_1$$

$$x_{12} + x_{22} + x_{33} = 2000 + 10a_2$$

$$x_{13} + x_{23} + x_{33} = 1000 + 10a_3$$

2)

$$x_{11} + x_{12} + x_{13} \leq 5000$$

$$x_{21} + x_{22} + x_{23} \leq 5000$$

$$x_{31} + x_{32} + x_{33} \leq 5000$$

3)

$$\sum_{i=1}^3 \sum_{j=1}^3 x_{ij} \leq 14000$$

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Average octane level = (total octane value) / number of barrels

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$$\frac{12x_{11} + 6x_{21} + 8x_{31}}{x_{11} + x_{21} + x_{31}} \geq 10$$

Is this linear? **YES** It is equivalent to

$$12x_{11} + 6x_{21} + 8x_{31} \geq 10(x_{11} + x_{21} + x_{31})$$

or

$$2x_{11} - 4x_{21} - 2x_{31} \geq 0$$

We can do the same for all the constraints, yielding the final linear program:

Final LP

maximize $21x_{11} + 11x_{12} + x_{13} + 31x_{31} + 21x_{22} + 11x_{23} + 41x_{31} + 31x_{32} + 21x_{33} - a_1 - a_2 - a_3$
s.t.

$$x_{11} + x_{21} + x_{31} - 10a_1 = 3000 \quad (\text{demand for gas 1})$$

$$x_{12} + x_{22} + x_{32} - 10a_2 = 2000 \quad (\text{demand for gas 2})$$

$$x_{13} + x_{23} + x_{33} - 10a_3 = 1000 \quad (\text{demand for gas 3})$$

$$x_{11} + x_{12} + x_{13} \leq 5000 \quad (\text{limit on crude 1})$$

$$x_{21} + x_{22} + x_{23} \leq 5000 \quad (\text{limit on crude 2})$$

$$x_{31} + x_{32} + x_{33} \leq 5000 \quad (\text{limit on crude 3})$$

$$x_{11} + x_{21} + x_{31} + x_{12} + x_{22} + x_{32} + x_{13} + x_{23} + x_{33} \leq 14000 \quad (\text{limit on total production})$$

$$2x_{11} - 4x_{21} - 2x_{31} \geq 0 \quad (\text{l.b. on octane of gas 1})$$

$$4x_{12} - 2x_{22} \geq 0 \quad (\text{l.b. on octane of gas 2})$$

$$6x_{13} + x_{33} \geq 0 \quad (\text{l.b. on octane of gas 3})$$

$$-.005x_{11} - .01x_{21} - .02x_{31} \leq 0 \quad (\text{u.b. on sulphur in gas 1})$$

$$.015x_{12} + .001x_{32} \leq 0 \quad (\text{u.b. on sulphur in gas 2})$$

$$-.005x_{13} - .01x_{23} + .02x_{33} \leq 0 \quad (\text{u.b. on sulphur in gas 3})$$

What are the decisions :

- The amount of crude i in gas j : x_{ij} .
- How much to advertise gas j : a_j .

Solution

$$x = \begin{pmatrix} 2222.22 & 2111.11 & 666.67 \\ 444.44 & 4222.22 & 333.34 \\ 333.33 & 3166.67 & 0 \end{pmatrix}$$

$$a = (0, 750, 0)$$

objective $z = 287500$