IEOR 3608

Introduction to Mathematical Programming

Midterm 1

Fall, 2002

October 9, 2002, 1:10 pm

This is a 75 minute exam. Be sure to show your work for partial credit.

Question	Points	Total
1		15
2		20
3		25
4		20
Total		80
Extra Credit		

Name: _____

The backs of the pages are for scratch work. We will not look at the backs unless you **explicitly** ask us to.

1. Short Answer [15 POINTS total] For each question, give a short answer and explanation.

a) **[5 POINTS]** My client describes an automobile manufacturing problem and I formulate a linear program that decides how many cars to build in order to maximize profit. Suppose that when I solve the linear program, the solution is unbounded. What can I conclude?

b) **[5 POINTS]** Suppose that I solve the linear program for the automobile manufacturing problem and find that the optimal solution is to manufacture 710.3 cars each day. What can I conclude about the validity of the model?

c) **[5 POINTS]** Would your answer to part b) change if the optimal solution is to manufacture .5 cars each day? Why or why not?

2) [**20 POINTS**] Use the simplex method to find an optimal solution to the following linear program. You must use the simplex method to receive credit: (You cannot just use a graphical method. You must solve this algebraically, using either the method from class, or the method from the book.) Show all your work!

$$
\begin{array}{lllll}
\text{maximize} & x_1 & -x_2 & & \\
\text{subject to} & & & & \\
 & x_1 & +x_2 & \geq & 1 \\
 & 2x_1 & +x_2 & \leq & 4 \\
 & x_1, x_2 & & \geq & 0
\end{array}
$$

3) [**25 POINTS**]

a) [**15 POINTS**] The New York Yankees have realized that their current strategy of hiring seemingly talented baseball players is leading nowhere. They decide instead to hire recent Columbia graduates in either "Industrial Engineering" (IE) or "Operations Research" (OR). They plan to hire a number of people immediately to work during the next 2 years. The people with IE degrees will be paid $50000 per year, while the people with OR degrees will be paid $60000 per year. The Yankees realize that their needs will be changing and that in the first year, they have 100 jobs that are appropriate for IEs and 80 jobs that are appropriate for ORs, but in the second year they have 60 jobs that are appropriate for IEs and 100 jobs that are appropriate for ORs.

The Yankees have no loyalty to their employees. They will hire people at the beginning of year 1. But, at the end of year one, they will choose to fire some of those people. If a person is fired, they must be given 20% of their salary as severance pay. They only want people to work in jobs for which they are trained, and therefore an IE can only perform an IE job and an OR can only perform an OR job.

Formulate (but do not solve) a linear program that hires a sufficient number of people at minimum cost to the Yankees. Be sure to explain, in words, what each of your decision variables represents.

b) [**10 POINTS**] The Yankees realize that firing many employees has a hidden cost. They therefore decide that they will hire people for 2 years, and commit to keeping them employed for the entire two years. The Yankees also decide to study, more carefully, the requirements at Columbia and realize that there is significant overlap between the IE and OR degrees. Therefore, they conclude that an IE can be placed in an OR job, but can only do the 1/2 the amount of work. In other words it takes 2 IEs to do an OR job. Similarly, it takes 2 ORs to do an IE job. People can shift jobs after one year, e.g. an OR can do an OR job in year 1 and an IE job (at 1/2 efficiency) in year 2.

Formulate (but do not solve) a linear program that hires a sufficient number of people at minimum cost to the Yankees. Be sure to explain, in words, what each of your decision variables represents.

4) **[20 POINTS]** The following linear program is a maximization problem. Two of the coefficients are denoted by A and B. You will be asked to give values of A and B so that the linear program satisfies certain conditions. **For each part give your answer and a brief explanation. An explanation must be given for full credit**

$$\begin{aligned} z &= 1 - 5x_1 + Ax_2 \\ s_1 &= 2 + 2x_1 - 4x_2 - 3x_3 \\ s_2 &= B - 4x_1 - 2x_2 - x_3 \end{aligned}$$

a) **[5 POINTS]** Give one pair of values for A and B for which the optimal solution to this linear program has objective value of 1.

b) **[5 POINTS]** Give one pair of values for A and B for which the next pivot of the simplex method could have x_2 entering and s_2 leaving.

c) **[5 POINTS]** Give one pair of values for A and B for which the linear program is degenerate.

d) **[5 POINTS]** What would it mean if B were -1?

Extra Credit – ONLY DO THIS AFTER YOU HAVE COMPLETED EVERYTHING ELSE Redo Question 5, parts a, b, and c, describing *all* pairs of values which answer each of the questions.