Branch and Bound for $1|r_j|L_{\max}$

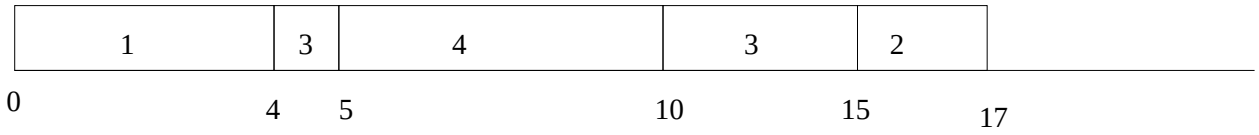
Branch and Bound (for minimization)

- **Set a variable and branch**
- **Compute lower bounds at nodes by solving a relaxed problem.**
- **Use lower bounds to prune tree. A lower bound at a node is a lower bound on all children.**
- **Three ways to prune:**
  1. **If a node is infeasible**
  2. **If a node has a lower bound that is larger than a candidate solution**
  3. **If a node has a value that is larger than a candidate solution.**

Example

j	r_j	p_j	d_j
1	**0**	**4**	**8**
2	**1**	**2**	**12**
3	**3**	**6**	**11**
4	**5**	**5**	**10**

To compute a lower bound we use a polynomially solvable relaxation, $1|r_j, \textbf{pmtn}|L_{\max}$**. This is solved by preemptive EDD.**



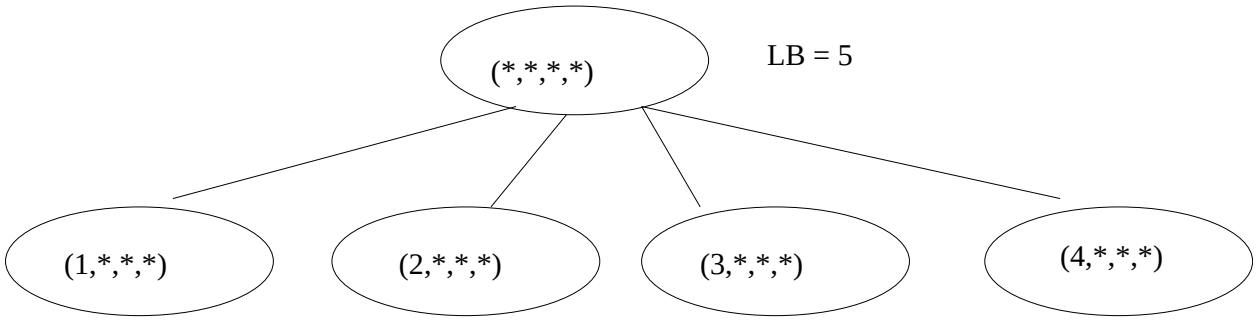
$$L_1 = -4, L_4 = 0, L_3 = 4, L_2 = 5.$$

$$L_{\max} = 5.$$

This means that for the real problem $L_{\max} \geq 5$**.**

Begin branching

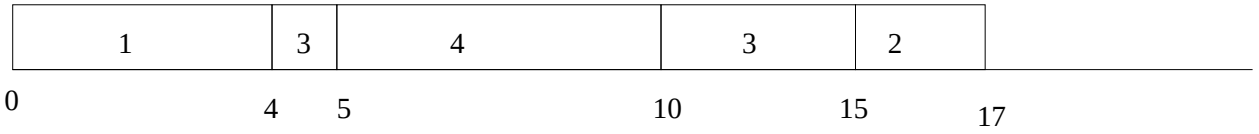
Branch on the choice of first job



Now for each branch we compute a new lower bound.

Computing lower bounds

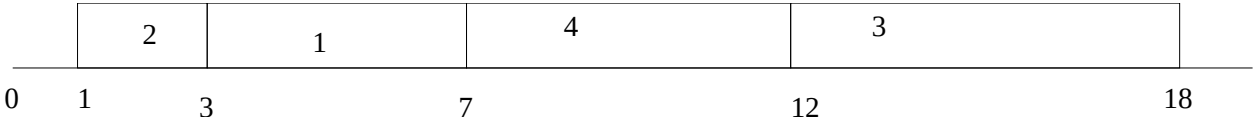
1 first. Same as original lowerbound



$$L_1 = -4, L_4 = 0, L_3 = 4, L_2 = 5.$$

$$L_{\max} = 5.$$

2 first.

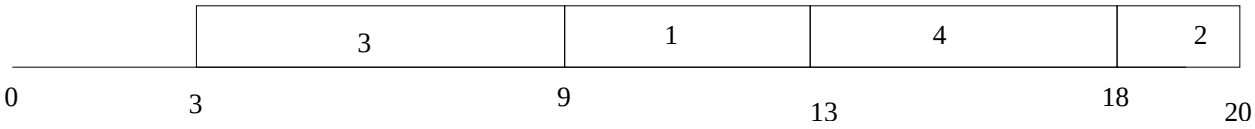


$$L_2 = -9, L_1 = -1, L_4 = 2, L_3 = 7.$$

$$L_{\max} = 7.$$

Note that this is an actual solution.

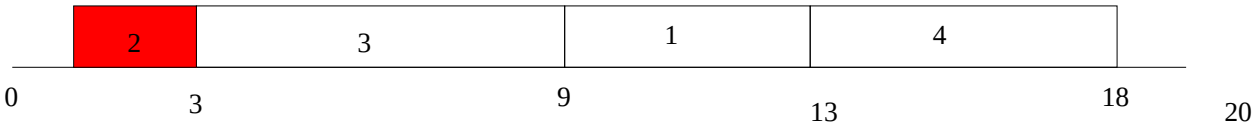
3 first.



$$L_3 = -2, L_1 = 5, L_4 = 8, L_2 = 8.$$

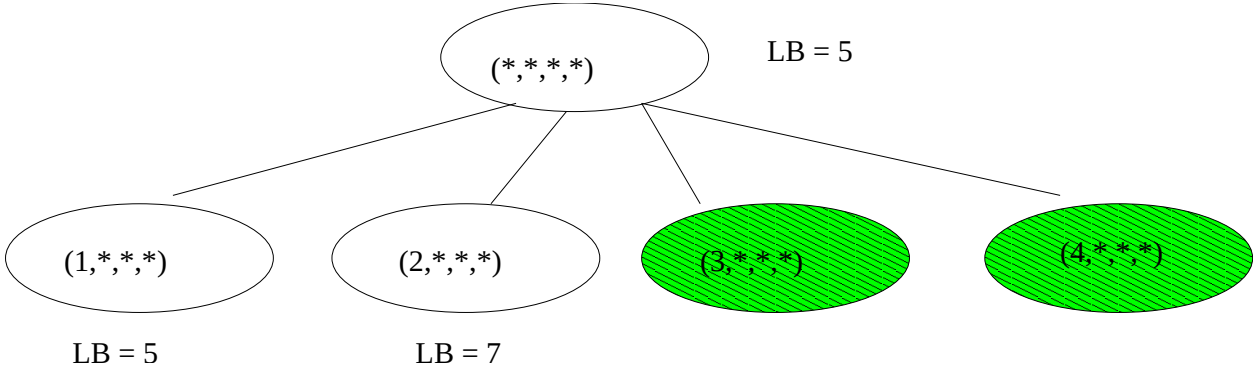
$$L_{\max} = 8.$$

But, this node can actually be pruned. It is clearly dominated by the following schedule

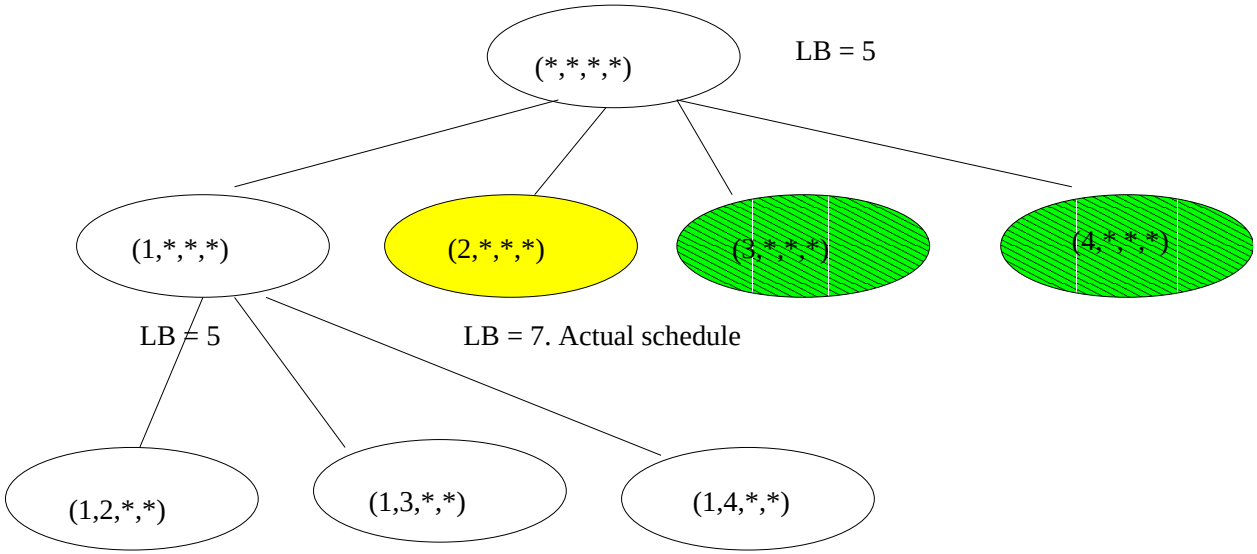


In other words, we know that starting with node three is a bad idea. For the same reason, we can prune starting at node 4.

Current tree

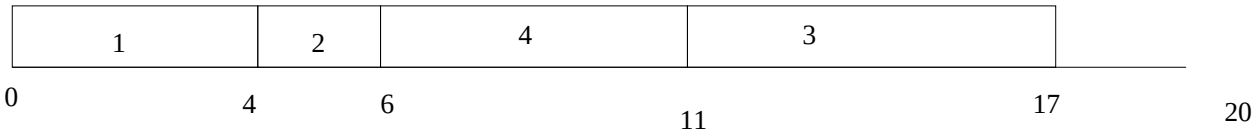


Choose a node to expand further. Can only choose 1.



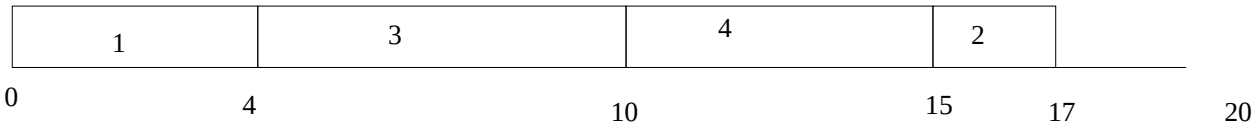
Now, we check the lower bounds at the nodes by solving the preemptive EDD schedule.

1,2 first.



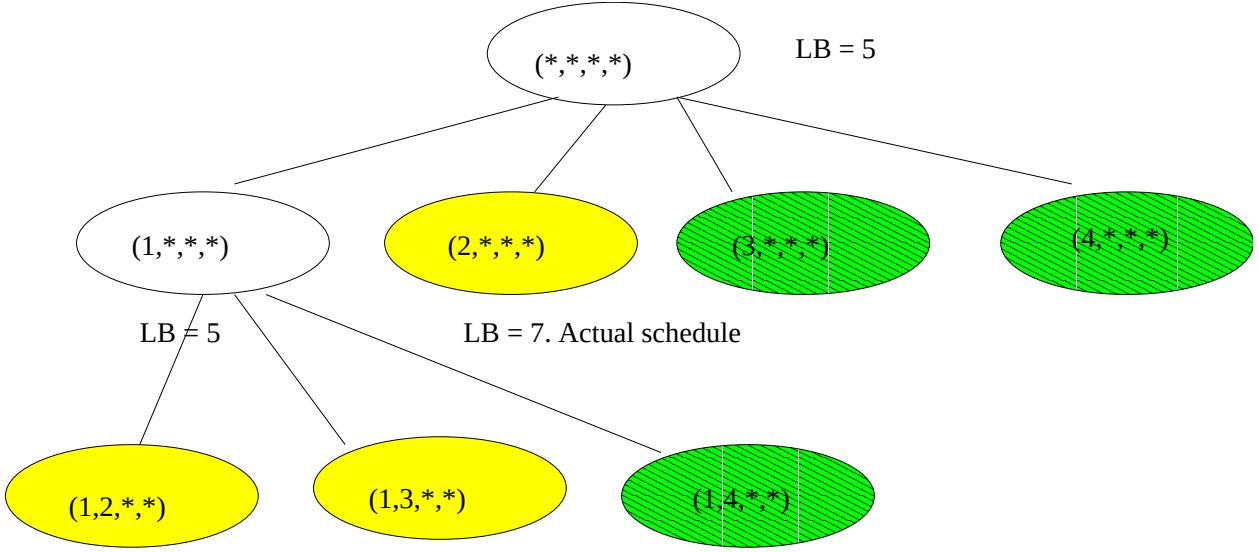
This is a real schedule with $L_{\max} = 6.$ **.**

1,3 first.



This is a real schedule with $L_{\max} = 5.$ **.**

We can actually stop now, because we have a schedule that matches our lower bound.


(*,*,*,*)
LB = 5
(1,*,*,*)
(2,*,*,*)
(3,*,*,*)
(4,*,*,*)
LB = 5
LB = 7. Actual schedule
(1,2,*,*)
(1,3,*,*)
(1,4,*,*)
Actual schedule.
LB = 6
Actual schedule.
LB = 5
can’t be better than 5.
THIS IS OPTIMAL