An algorithm for $1||\Sigma T_j$.

Use dynamic programming, from the following formulas:

- Let $V(J,t)$ be the optimal schedule for job set J starting at time t.
- Let $J(j,l;k)$ be the set of job with indices between j and l and which have processing time less than p_k. Note that k is excluded from this set.

Recursion:

$$V(\emptyset, t) = 0.$$

$$V(\{j\}, t) = \max(0, t + p_j - d_j)$$

$$\begin{aligned} V(J(j,l;k),t) &= \min_{\delta}\big(V(J(j,k'+\delta;k'),t) \\ &+ \max(0, C_{k'}(\delta) - d_{k'}) \\ &+ V(J(k'+\delta+1,l,k'),C_{k'}(\delta))\big) \end{aligned}$$

where k' is the job with minimum processing time in $J(j,l;k)$.

Example

j	p_j	d_j
1	121	260
2	79	266
3	147	266
4	83	336
5	130	337

$$V(1,\ldots,5,0) =$$

$$\min\left\{\begin{array}{l} V(J(1,3,3),0)+81+V(J(4,5,3),347), \\ V(J(1,4,3),0)+164+V(J(5,5,3),430), \\ V(J(1,5,3),0)+294+V(\emptyset,560) \end{array}\right\}$$

- $J(1,3,3) = \{1,2\}$. These can be scheduled in order 1,2, with objective value 0.
- $J(1,4,3) = \{1,2,4\}$. We compute this recursively and get that we can schedule $(1,2,4)$ with objective value 0.
- $J(1,5,3) = \{1,2,4,5\}$. We compute this recursively and get the schedule $(1,2,4,5)$ with value 347.

- $J(4,5,3) = \{4,5\}$. $V(J(4,5,3), 347)$ is the optimal schedule for 4 and 5 starting at time 347. This order is $(4,5)$ with value $94 + 223 = 317$.
- $J(5,5,3) = \{5\}$. $V(J(5,5,3), 430)$ is the optimal way to schedule job 5 starting at time 430.

so we get

$$\min \left\{ \begin{array}{l} 0 + 81 + 317 \\ 0 + 164 + 223, \\ 76 + 294 + 0 \end{array} \right\} = 370$$

So optimal schedule is $1, 2, 4, 5, 3$

Recursion for $J(1, 4, 3)$

$k' = 4,$

$$J(1, 4, 3) = \min \left\{ V(J(1, 4, 4), 0) + 0 + V(\emptyset, 283) \right\}$$

Recursion for $J(1, 5, 3)$

$k' = 5$

$$J(1, 5, 3) = \min \left\{ V(J(1, 5, 5), 0) + 76 + V(\emptyset, 413 \right\}$$