Tanker Scheduling

Ships have:

- capacity
- draught (minimum depth to float)
- range of speeds and fuel consumption
- location and available time

Ports have:

- weight limits
- draught
- other physical restrications
- government restrictions

Tanker Scheduling (cont)

Cargo has

- type
- load port
- destination prot
- time constraints
- load and unload times

A company will own ships and may rent ships. It is more expensive to rent ships.

Objective: minimize cost

- operarting costs for company shipos
- charter rates
- fuel costs
- port charges

Formulation

Notation:

Parameters

- n **- number of cargoes**
- T **- number of company owned tankers**
- p **- number of ports**

plus data for all of the above.

Compute

- S_i **- the set of possible schedules for ship** i**.** $a_{ij}^l = 1$ **if under schedule** l **ship** i **trnasports cargo** J**.**
- C_j^* **is amount paid to transport cargo** j **on a ship that is not company owned.**
- c_i^l **- incremental cost of operatins a company-owned shipo** i **under schedule** l **versus keeping shipo** i **idle.**
- **Compute the profit for operationg ship** i **according to schedule** $l - \pi_i^l = \Sigma_{j=1}^n a_{ij}^l c_j^* - c_i^l$**.**

Formulation

Decision variable: x_i^l **if ship** i **follows schedule** l**.**

Formulation

$$
\begin{aligned}
\textbf{maximize } & \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l \\
\textbf{subject to} & \\
\sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l & \leq 1 \quad j = 1, \ldots, n \\
\sum_{l \in S_i} x_i^l & \leq 1 \quad i = 1, \ldots, T \\
x_i^l & \in \{0, 1\} \quad l \in S_i, i = 1, \ldots, T
\end{aligned}
$$

Solution **Set packing. Use branch and bound.**

Example

- **3 ships**
- **12 cargoes**

Analysis of the data show that each of the ships has five feasible schedules:

Schedules	a_{1j}^1	a_{1j}^2	a_{1j}^3	a_{1j}^4	a_{1j}^5	a_{2j}^1	a_{2j}^2	a_{2j}^3	a_{2j}^4	a_{2j}^5	a_{3j}^1	a_{3j}^2	a_{3j}^3	a_{3j}^4	a_{3j}^5
cargo 1	1	0	0	1	1	0	1	0	0	0	0	0	0	1	0
cargo 2	1	0	0	0	0	1	0	0	0	0	0	1	0	1	1
cargo 3	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0
cargo 4	0	1	1	1	0	1	0	1	0	0	0	0	0	0	0
cargo 5	1	1	0	0	0	0	0	0	1	0	0	0	1	0	1
cargo 6	0	0	0	1	1	0	1	0	0	1	1	0	0	0	0
cargo 7	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
cargo 8	0	1	0	0	0	0	1	0	1	1	1	0	0	0	0
cargo 9	0	0	1	0	0	0	1	0	0	1	1	1	1	0	0
cargo 10	0	1	0	0	0	1	0	0	0	0	1	1	0	0	0
cargo 11	0	0	0	0	0	0	1	1	0	0	0	1	1	1	0
cargo 12	0	0	0	1	0	0	0	0	0	0	1	0	1	1	1

Costs

Charter cost for transporting a particular cargo by charter:

Cargo	1	2	3	4	5	6	7	8	9	10	11	1
Charter Costs	1429	1323	1208	512	2173	2217	1775	1885	2468	1928	1634	7

Operating costs of the tankers under each one of the schedules is also given:

Schedule l	1	2	3	4	5
cost of tanker 1 (c_1^l)	5608	5033	2722	3505	3996
cost of tanker 2 (c_2^l)	4019	6914	4693	7910	6866
cost of tanker 3 (c_3^l)	5829	5588	82824	3338	4715

We can compute the profit for each schedule

Schedule l	1	2	3	4	5
profit of tanker 1 (π_1^l)	-733	1465	1466	1394	858
profit of tanker 2 (π_2^l)	1629	834	1113	-869	910
profit of tanker 3 (π_3^l)	1525	1765	-1268	1789	1297

IP

Now we can give an IP

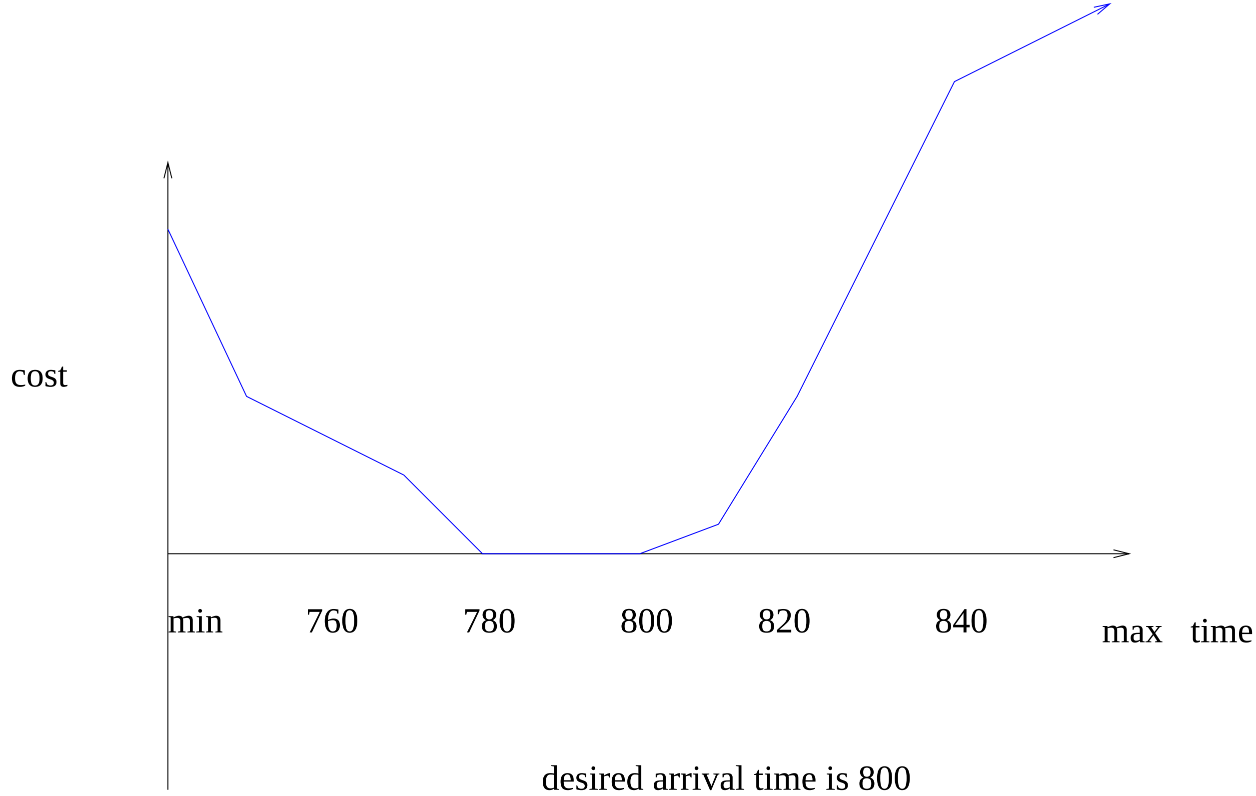
$$
\begin{aligned}
\textbf{maximize} -733x_1^1 + 1465x_1^2 + 1466x_1^3 + 1394x_1^4 + 858x_1^5 \\
+1629x_2^1 + 834x_2^2 + 1113x_2^3 + -869x_2^4 + 910x_2^5 \\
+1525x_3^1 + 1765x_3^2 + -1268x_3^3 + 1789x_3^4 + 1297x_3^5 \\
\textbf{subject to} \\
x_1^1 + x_1^4 + x_1^5 + x_2^2 + x_3^4 &\le 1 \\
x_1^1 + x_2^2 + x_3^2 + x_3^4 + x_3^5 &\le 1 \\
x_1^3 + x_1^5 + x_2^4 + x_2^5 &\le 1 \\
x_1^2 + x_1^3 + x_1^4 + x_2^1 + x_2^3 &\le 1 \\
x_1^1 + x_1^2 + x_2^4 + x_3^3 + x_3^5 &\le 1 \\
x_1^4 + x_1^5 + x_2^2 + x_2^5 + x_3^1 &\le 1 \\
x_2^3 + x_2^4 + x_3^5 &\le 1 \\
x_1^2 + x_2^1 + x_2^3 + x_2^4 + x_2^5 &\le 1 \\
x_1^3 + x_2^2 + x_2^5 + x_3^1 + x_3^2 + x_3^3 &\le 1 \\
x_1^2 + x_2^1 + x_3^1 + x_3^2 &\le 1 \\
x_2^2 + x_2^3 + x_3^2 + x_3^3 + x_3^4 &\le 1 \\
x_1^4 + x_3^1 + x_3^3 + x_3^4 + x_3^5 &\le 1 \\
\\
x_1^1 + x_1^2 + x_1^3 + x_1^4 + x_1^5 &\le 1 \\
x_2^1 + x_2^2 + x_2^3 + x_2^4 + x_1^5 &\le 1
\end{aligned}
$$

$$x_3^1 + x_3^2 + x_3^3 + x_3^4 + x_3^5 \leq 1$$
$$x_i^l \in \{0, 1\}$$

Optimal solution **Schedule 3 for ship 1, schedule 4 for ship 3. Ship 2 remains idle. Cargoes 5,6,7,8,10 are transported by charters. Value = 3255.**

Train timetabling

- **One track with many stations (think 1/9 subway line or commuter rail).**
- **Trains can pass at statations but not between stations.**
- **Stations are numbered 0 to L.**
- **Tracks are numbered 1 to $L+1$.**
- **Track i connectes station $j-1$ with j.**
- **Time is measured in minutes (1 to 1440).**
- **You are given preferred arrival, departure, travel, and holdover times for each train at each station.**
- **There is a piecewise linear function measuring the cost (lost revenue) of deviating from the desired time.**


cost
min
760
780
800
820
840
max
time
desired arrival time is 800

IP

Variables

- y_{ij} = time train i enters link j (leaves station $j-1$)
- z_{ij} = time train i exits line j (arrives at station j)

We compute

- $\tau_{ij} = z_{ij} - y_{ij}$ (travel time of train i in link j)
- $\delta_{ij} = y_{i,j+1} - z_{ij}$ (dwelling time of train i in station j)

We are given costs for each of these quantities:

- $c^a_{ij}(z_{ij})$ - costs for train i arriving at station j
- $c^d_{ij}(y_{ij})$ - costs for train i departing from station j
- $c^\tau_{ij}(\tau_{ij})$ - costs for travel time of train i in link j
- $c^\delta_{ij}(\delta ij)$ - costs for travel time of train i dwelling in station j.

Each of these costs is piecewise linear, and we are given min and max values. Also, minimum and maximum headway values H

T is the set of possible trains.

Variable: $x_{hij} = 1$ is train h immediately precedes train i on link j.

IP

$$\textbf{minimize} \ \sum_{i\in T}\sum_{j=1}^{L}\left(c^a_{ij}(z_{ij}) + c^d_{i,j-1}(y_{ij}) + c^\tau_{ij}(\tau_{ij})\right) + \sum_{i\in T}\sum_{j=1}^{L-1}(c^\delta_{ij}(\delta ij))$$

subject to

$$\begin{array}{rcl}
y_{ij} & \geq y^{\min}_{ij} & i \in T, j = 1,\ldots,L \\
y_{ij} & \leq y^{\max}_{ij} & i \in T, j = 1,\ldots,L \\
z_{ij} & \geq z^{\min}_{ij} & i \in T, j = 1,\ldots,L \\
z_{ij} & \leq z^{\max}_{ij} & i \in T, j = 1,\ldots,L \\
\tau_{ij} & = z_{ij} - y_{ij} & i \in T, j = 1,\ldots,L \\
\tau_{ij} & \geq \tau^{\min}_{ij} & i \in T, j = 1,\ldots,L \\
\tau_{ij} & \leq \tau^{\max}_{ij} & i \in T, j = 1,\ldots,L \\
\delta_{ij} & = y_{i,j+1} - z_{ij} & i \in T, j = 1,\ldots,L \\
\delta_{ij} & \geq \delta^{\min}_{ij} & i \in T, j = 1,\ldots,L-1 \\
\delta_{ij} & \leq \delta^{\max}_{ij} & i \in T, j = 1,\ldots,L-1 \\
y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M & \geq H^d_{hij} & i \in T, j = 1,\ldots,L \\
z_{ij} - z_{hj} + (1 - x_{hik})M \geq H^a_{hij} & i \in T, j = 1,\ldots,L & \\
\sum_{h\in\{T-i\}} x_{hij} & = 1 & i \in T, j = 1,\ldots,L \\
x_{hij} & \in \{0,1\} &
\end{array}$$

Solution

Can solve using heuristic similar to shifting bottleneck heuristic. Set one train (by importance) and resolve LP relaxation.