

# Branch and Bound

- One way of dealing with an NP-complete problem.
- Not polynomial.
- Complete enumeration in the worst case, but uses a clever idea to cut off much of the search space

## Branch and Bound for minimizing (0-1) IPs

- Solve an LP relaxation
- Choose a variable  $x_i$  with value strictly between 0 and 1.
- Branch on whether  $x_i = 0$  or  $x_i = 1$ .
- Bound
  - Generate feasible solutions to the IP
  - At each node generate a lower bound (using the LP) on the solutions in that branch.
  - If the lower bound on a branch is better than a feasible solution, kill that branch.
- When there is no more branching to be done, stop.

# Branch and Bound

## Combinatorial Branch and Bound for a minimization problem

- Enumerate possible values for a decision.
- Branch on all the possibilities
- Bound
  - Generate feasible solutions to original problem.
  - At each node generate a lower bound (using a relaxation) on the solutions in that branch.
  - If the lower bound on a branch is better than a feasible solution, kill that branch.
- When there is no more branching to be done, stop.

## Branch and Bound for $1|r_j|L_{\max}$

- Branch on “which job runs in position  $i$  .
- Use  $1|r_j, pmtn|L_{\max}$  as a lower bound

### Example

$j$	$r_j$	$p_j$	$d_j$
1	0	4	8
2	1	2	12
3	3	6	11
4	5	5	10