Scheduling Notation

We will give the typical notation used in the course

Jobs

- **Number:** n
- **Typical Index:** j
- **Features:**
  - **processing time:** p_j **or** p_{ij}
  - **release date:** r_j
  - **deadline or due date:** d_j
  - **weight** w_j

Machines

- Number: m
- Typical Index: i
- Possible Environments:
  - **1** : one machine
  - **P,Pm**: parallel (identical machines)
  - **Q,Qm**: related machines (diffent speeds)
  - **R,Rm**: unrelated machines (processing time depends on job and machine)
  - Shop Environments
    - **J**: job shop – each job has linear constraints among its task
    - **F**: flow shop – each job has the same linear constraints among its task
    - **O**: open shop – no constraints among tasks

Constraints

We give some examples here:

- r_j: **release date**
- **pmtn: preeeption**
- **prec: precedence constraints**
- s_{jk}: **sequence dependent set up times**
- **bkdwn: machines may breakdown**
- **block: limited buffer size**

Objectives

- A schedule designates which job runs on which machine at each time. It therefore assigns a completion time C_j to each job j
- We evaluate a job by some function of C_j and the other parameters of job, e.g.
  * Lateness: $L_j = C_j - d_j$
  * Tardiness: $T_j = \max L_j, 0$
  * Unit Cost: $U_j = 1$ if $C_j > d_j$ and 0 otherwise
  * Flow (Response) Time: $F_j = C_j - r_j$
  * Idle Time: $I_j = C_j - r_j - p_j$
  * Stretch: $S_j = (C_j - r_j)/p_j$
- We then evaluate a schedule by some function of the job functions, usually a minimization of a
  * sum
  * weighted sum
  * discounted weighted sum
  * maximum (We use $X_{\max}$ as shorthand for $\max_j X_j$.

3 field notation

- machines — constraints — objective
- Default is no preemption Examples:
  - $P||C_{\max}$ – parallel identical machines, minimize the schedule length (makespan)
  - $1|\text{prec}, \text{pmtn}|\sum w_j C_j$ - one machine, precedence constraints and preemption, minimize the sum of weighted completion times
  - $P\infty|\text{prec}|C_{\max}$ - project scheduling
  - $Jm|nowait|C_{\max}$ - nowait job shop scheduling, minimize makespan
  - $1|\text{pmtn}|\sum w_j T_j$ - one machine, preemption, minimum weighted tardiness