Scheduling Notation

We will give the typical notation used in the course
Jobs

- Number: \( n \)
- Typical Index: \( j \)
- Features:
  - processing time: \( p_j \) or \( p_{ij} \)
  - release date: \( r_j \)
  - deadline or due date: \( d_j \)
  - weight \( w_j \)
Machines

– Number:  \( m \)
– Typical Index:  \( i \)
– Possible Environments:
  * 1 : one machine
  * \( P,Pm \): parallel (identical machines)
  * \( Q,Qm \): related machines (different speeds)
  * \( R,Rm \): unrelated machines (processing time depends on job and machine)
* Shop Environments
  * J: job shop – each job has linear constraints among its task
  * F: flow shop – each job has the same linear constraints among its task
  * O: open shop – no constraints among tasks
Constraints

We give some examples here:

- \( r_j \): release date
- \( \text{pmtn} \): preemption
- \( \text{prec} \): precedence constraints
- \( s_{jk} \): sequence dependent set up times
- \( \text{bkdwn} \): machines may breakdown
- \( \text{block} \): limited buffer size
Objectives

- A **schedule** designates which job runs on which machine at each time. It therefore assigns a completion time \( C_j \) to each job \( j \).
- We evaluate a job by some function of \( C_j \) and the other parameters of job, e.g.
  * Lateness: \( L_j = C_j - d_j \)
  * Tardiness: \( T_j = \max L_j, 0 \)
  * Unit Cost: \( U_j = 1 \) if \( C_j > d_j \) and 0 otherwise
  * Flow (Response) Time: \( F_j = C_j - r_j \)
  * Idle Time: \( I_j = C_j - r_j - p_j \)
  * Stretch: \( S_j = (C_j - r_j)/p_j \)
- We then evaluate a schedule by some function of the job functions, usually a minimization of a
  * sum
  * weighted sum
  * discounted weighted sum
  * maximum (We use \( X_{\text{max}} \) as shorthand for \( \max_j X_j \).
3 field notation

- machines — constraints — objective
- Default is no preemption Examples:
  - $P||C_{\text{max}}$ — parallel identical machines, minimize the schedule length (makespan)
  - $1|\text{prec, pmtn}|\sum w_j C_j$ - one machine, precedence constraints and preemption, minimize the sum of weighted completion times
  - $P\infty|\text{prec}|C_{\text{max}}$ - project scheduling
  - $Jm|\text{nowait}|C_{\text{max}}$ - nowait job shop scheduling, minimize makespan
  - $1|\text{pmtn}|\sum w_j T_j$ - one machine, preemption, minimum weighted tardiness