Airline Crew Pairing Problem

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Agenda

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• Common column generation
• Advanced Column generation
• Implementation of Advanced Column generation
• Comparison
• Crew pairing considering robustness
• Row and column generation
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Summary

Topic

• Airline crew pairing problem
  = large-scale linear problem + integer programming problem

Our Project

1. Column generation
   1) Common column generation (Research paper 1)
   2) Advanced column generation (Research paper 2)

2. Application of column generation
   1) Crew pairing considering robustness (Research paper 3)
   2) Row and column generation
Research papers

Common column generation  (Research paper 1)

•  Airline crew scheduling.


Advanced  column generation (Research paper 2)

•  A new approach for crew pairing problems by column generation with and application to air transportation.


Application of column generation  (Research paper 3)

•  Solving a robust airline crew pairing problem with column generation.

Airline Crew Pairing Problem

Crew pairing problem

\[ \min \sum_{p \in P} c_p y_p \]

\[ \sum_{p : i \in p} y_p = 1 \quad i \in F \]

\[ y_p \in \{0, 1\} \quad p \in P \]

\[ c_p : \text{Cost of Pair} \]
\[ p : \text{Crew pairing} \]
\[ P : \text{Set of all feasible pairings} \]
\[ i : \text{Flight} \]
\[ F : \text{Set of flight segments to be covered} \]
\[ y_p : 1 \text{ if pairing } p \text{ is included in the solution} \]
\[ 0 \text{ otherwise} \]

Ex) When \( m = 10, n = 100 \)
\[ 2^{m+n} = 1.07 \times 10^{301} \]

large-scale integer liner problem \( \Pi \)

NP-hard
Column generation
Common column generation

Key idea of Column Generation

Since most of the variables will be non-basic and assume a value of zero in the optimal solution, only a subset of variables need to be considered.

Relaxation → Reduction of calculation → Solvable

Algorithm of Column Generation

Step 1: Make a new combination of pairs (by adding a pair)
Step 2: With the combination, create a linear problem (Relaxation)
Step 3: Solve the problem and find optimal solution for the problem
Step 4: Check reduced cost with the optimal and decide which pairs to keep (Reduced cost = negative)
   4-1: If no remaining pair → Step 5
   4-2: otherwise → Step 1 with adding a pair
Step 5: With the pairs adopted in Step 4, solve the integer problem.
Advanced Column generation

Concept

- Basic idea is same as normal column generation
- Difference from normal column generation is in creating new combination of pairs

Normal column generation

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>k Pair</td>
<td>k' + 1 Pair</td>
<td>K'' + 1 Pair</td>
</tr>
<tr>
<td>n - k Pair</td>
<td>n - (k+1) Pair</td>
<td>n - (k+2) Pair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
</tr>
</tbody>
</table>

Advanced column generation

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>k Pair</td>
<td>k Pair</td>
<td>k Pair</td>
</tr>
<tr>
<td>n - k Pair</td>
<td>l Pair</td>
<td>m Pair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n - (k + l + m) Pair</td>
</tr>
<tr>
<td></td>
<td></td>
<td>:</td>
</tr>
</tbody>
</table>
Implementation of Advanced Column generation

Data Condition

\[
\begin{align*}
\min \ C'x \\
\text{s.t. } \ Ax = 1
\end{align*}
\]

- $C'$: Cost vector (1000*1 matrix)
- $A$: Constraint matrix (10*1000 matrix)
- $x$: Object variable (1000*1 matrix)

PC Condition

- Intel(R) Core(TM) i5-2467M CPU 1.60 GHz
  1.60 GHz
- Memory: 4.00 GB
- OS: Windows 7 Home Premium
## Comparison

<table>
<thead>
<tr>
<th>Normal column generation</th>
<th>Advanced column generation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LP Relaxation ie Lower Bound</strong></td>
<td><strong>LP Relaxation ie Lower Bound</strong></td>
</tr>
<tr>
<td>1.2000</td>
<td>1.2000</td>
</tr>
<tr>
<td><strong>IP, Initial Opt Value</strong></td>
<td><strong>LP Relaxation, Initial Opt Value</strong></td>
</tr>
<tr>
<td>17</td>
<td>6.5000</td>
</tr>
<tr>
<td><strong>LP Relaxation, Opt Value with Selected Variables</strong></td>
<td><strong>LP Relaxation, Opt Value with Selected Variables</strong></td>
</tr>
<tr>
<td>6.5000</td>
<td>1.3333</td>
</tr>
<tr>
<td><strong>IP, Opt Value with Selected Variables</strong></td>
<td><strong>IP, Opt Value with Selected Variables</strong></td>
</tr>
<tr>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td><strong>Num of Selected Variables</strong></td>
<td><strong>Num of Selected Variables</strong></td>
</tr>
<tr>
<td>132</td>
<td>490</td>
</tr>
<tr>
<td><strong>Calc Time</strong></td>
<td><strong>Calc Time (sec)</strong></td>
</tr>
<tr>
<td>646.7000</td>
<td>74.4860</td>
</tr>
</tbody>
</table>
Application of column generation
Crew pairing considering robustness

Case

- Extra flight is required by unexpected problem (Bad weather, problem with an Aircraft)

Definition of Robustness

- Type A: Two pairings are selected and swapped to cover an extra flight.
- Type B: One pairing with sufficient connection time between two consecutive flights is modified to cover an extra flight.

Formula (constraint)

\[ \sum_{(p,q) \in P_A(k)} x_{(p,q)}^k \geq \alpha_k \quad k \in K \]

- \( K \): Set of all extra flights
- \( k \): Extra flight
- \( P_A(k) \): Set of pairing pairs that provide a type A solution for extra flight \( k \)
- \( \alpha_k \): Minimum number of type A solution that are required in the pairing solution
- \( p,q \): Pairing
Row and column generation

Additional constraints

Row and column generation
Normal column generation

Phase 1

\[
\begin{bmatrix}
k & \text{Pair} \\
\end{bmatrix}
\begin{bmatrix}
n-k & \text{Pair} \\
\end{bmatrix}
\]

Phase 2

\[
\begin{bmatrix}
k' + 1 & \text{Pair} \\
\end{bmatrix}
\begin{bmatrix}
n - (k+1) & \text{Pair} \\
\end{bmatrix}
\]

Column and row generation

Phase 1

\[
\begin{bmatrix}
n-k & \text{Pair} \\
\end{bmatrix}
\]

Phase 2

\[
\begin{bmatrix}
n - k - k' & \text{Pair} \\
\end{bmatrix}
\]

More difficult
1. **Column generation**
   1) Common column generation (Research paper 1)
   2) Advanced column generation (Research paper 2)

   Advanced column generation gives better solution

2. **Application of column generation**
   1) Crew pairing considering robustness
   2) Row and column generation

   New approach works crew pairing with robustness
Thank You
Appendix

Robustness Type A

(a) Original pairings $p$ and $q$.

(b) Pairings $p$ and $q$ are partially swapped.

Robustness Type B

(a) Original pairing $p$.

(b) Pairing $p$ is modified to cover extra flight $k$.

(c) Original pairing $p$.

(d) Pairing $p$ is modified to cover extra flight $k$.

Figure 2: Extra flight $k$ is inserted into pairing $p$ (type B solution).