Stochastic Scheduling

Models Real World Uncertainty

- **processing times**
- **arrivals**
- **machine availability**
- ...

Our Model:

- **Distribution over job data known in advance.**
- **Realization only known when job arrives/completes or when it can be inferred.**

Example:

$$p_j = \begin{cases} 1 & \Pr = 1/2 \\ 3 & \Pr = 1/2 \end{cases}$$

After 1 unit of time, if the job doesn't complete, we know that it will take 3 units.

Example

$$p_1 = \begin{cases} 1 & \Pr = 1/2 \\ 9 & \Pr = 1/2 \end{cases}$$

$$p_2 = \begin{cases} 4 & \Pr = 1/4 \\ 6 & \Pr = 1/2 \\ 8 & \Pr = 1/4 \end{cases}$$

Problem: $1||\sum C_j$

Question: **What is the right algorithm? Is there still a simple ordering rule**

Comparing random variables

- **Density Function:** $f(x)$
- **Distribution Function:** $F(x) = P(X \leq t) = \int_0^t f(x)dx$

Definitions of $X_1 \succeq X_2$

- **Larger in Expectation:** $E(X_1) \geq E(X_2)$
- **Stochastically larger:** $\forall t : P(X_1 > t) \geq P(X_2 > t)$
- **Almost surely larger:** $P(X_1 \geq X_2) = 1$

Another example, $P||C_{\max}$

Case 1: $p_1 = p_2 = 1$

Case 2: $p_1 = 1$

$$p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 3:

$$p_1 = p_2 = \begin{cases} 0 & \Pr = 1/2 \\ 2 & \Pr = 1/2 \end{cases}$$

Case 4: p_1, p_2 **both uniform in** $[0, 2]$**.**

Objective Values

1. $C_{\max} = 1$

2. $C_{\max} = 3/2$

3. $C_{\max} = 3/2$

4. $C_{\max} = 4/3$

Different Models of Stochastic Scheduling

Models of Knowledge

- **static: Choose order of jobs based on distribution only**
- **dynamic: Choose order of jobs based on knowledge gained when running**

Also consider Preemption vs. Non-preemption

Example:

- $1||\sum U_j$
- **3 jobs with same distribution:**

$$p_j = \begin{cases} 2 & \Pr = 1/2 \\ 8 & \Pr = 1/2 \end{cases}$$

$$d_j = \begin{cases} 1 & \Pr = 1/2 \\ 5 & \Pr = 1/2 \end{cases}$$

What is the expected objective value for:

- **static non-preemptive**
- **dynamic non-preemptive**
- **dynamic preemptive**

Another Example

- **Problem:** $1|\mathrm{pmtn}|\sum C_j$
- **Jobs:**

$$p_1 = \begin{cases} 1 & \Pr = 1/2 \\ 3 & \Pr = 1/2 \end{cases}$$

$$p_2 = \begin{cases} 2 & \Pr = 1/2 \\ 4 & \Pr = 1/2 \end{cases}$$

$$p_3 = \begin{cases} 1 & \Pr = 1/2 \\ 7 & \Pr = 1/2 \end{cases}$$