Scheduling Notation

We will give the typical notation used in the course

Jobs

- **Number:** n
- **Typical Index:** j
- **Features:**
  - **processing time:** p_j **or** p_{ij}
  - **release date:** r_j
  - **deadline or due date:** d_j
  - **weight** w_j

Machines

- **Number:** m
- **Typical Index:** i
- **Possible Environments:**
  - **1 : one machine**
  - **P,Pm: parallel (identical machines)**
  - **Q,Qm: related machines (different speeds)**
  - **R,Rm: unrelated machines (processing time depends on job and machine)**
  - **Shop Environments**
    - **J: job shop – each job has linear constraints among its task**
    - **F: flow shop – each job has the same linear constraints among its task**
    - **O: open shop – no constraints among tasks**

Constraints

We give some examples here:

- r_j: **release date**
- **pmtn**: **preemption**
- **prec**: **precedence constraints**
- s_{jk}: **sequence dependent set up times**
- **bkdwn**: **machines may breakdown**
- **block**: **limited buffer size**

Objectives

- A schedule designates which job runs on which machine at each time. It therefore assigns a completion time C_j to each job j
- We evaluate a job by some function of C_j and the other parameters of job, e.g.
  - Lateness: $L_j = C_j - d_j$
  - Tardiness: $T_j = \max\{L_j, 0\}$
  - Unit Cost: $U_j = 1$ if $C_j > d_j$ and 0 otherwise
  - Flow (Response) Time: $F_j = C_j - r_j$
  - Idle Time: $I_j = C_j - r_j - p_j$
  - Stretch: $S_j = (C_j - r_j)/p_j$
- We then evaluate a schedule by some function of the job functions, usually a minimization of a
  - sum
  - weighted sum
  - discounted weighted sum
  - maximum (We use $X_{\max}$ as shorthand for $\max_j X_j$).

3 field notation

- **machines — constraints — objective**
- **Default is no preemption Examples:**
  - $P||C_{\max}$ **– parallel identical machines, minimize the schedule length (makespan)**
  - $1|\text{prec}, \text{pmtn}|\sum w_j C_j$ **- one machine, precedence constraints and preemption, minimize the sum of weighted completion times**
  - $P\infty|\text{prec}|C_{\max}$ **- project scheduling**
  - $Jm|\text{nowait}|C_{\max}$ **- nowait job shop scheduling, minimize makespan**
  - $1|\text{pmtn}|\sum w_j T_j$ **- one machine, preemption, minimum weighted tardiness**