Average Completion Time on Multiple Machines

$P||\sum C_j$

j	p_j
A	**1**
B	**3**
C	**4**
D	**5**
E	**6**
F	**9**
G	**12**
H	**20**
I	**50**
J	**60**

What is the right algorithm?

Average Completion Time on Multiple Machines

- $P||\sum C_J$ – **SPT is optimal.**
- $P||\sum w_j C_j$ – **Is WSPT optimal?**

Example

j	w_j	p_j
1	**1**	**1**
2	**1**	**1**
3	**100**	**99**

Average Completion Time on Multiple Machines

- $P||\sum C_J$ – **SPT is optimal.**
- $P||\sum w_j C_j$ – **Is WSPT optimal?**

Example

j	w_j	p_j
1	**1**	**1**
2	**1**	**1**
3	**100**	**99**

- $P||\sum w_j C_j$ **is NP-complete.**
- **WSPT is a** $(1+\sqrt{2})/2$ **-approximation for** $P||\sum w_j C_j$

$R||\Sigma C_j$

- **Can be solved as a matching problem.**
- **Left side node for each job** j
- **Right hand side node for the** k **th from last job on machine** i

Example

	J_1	J_2	J_3	J_4
M_1	**6**	**4**	∞	**3**
M_2	**7**	**5**	**2**	**3**
M_3	**3**	**8**	**5**	**3**

LP for the matching problem

Variable $x_{ijk} = 1$ **if** j **is the** k **th from last job on** M_i

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{n} k p_{ij} x_{ijk}$$

s.t.

Each job runs

$$\sum_{i=1}^{m} \sum_{k=1}^{n} x_{ijk} = 1 \qquad j = 1 \ldots n$$

Each machine/slot has at most 1 job

$$\sum_{j=1}^{n} x_{ijk} \leq 1 \qquad i = 1 \ldots m; k = 1 \ldots n$$

$$x_{ijk} \in \{0, 1\} \quad i = 1 \ldots m; j = 1 \ldots n; k = 1 \ldots n$$

- **Note that the may be unforced idleness e.g.**

	J_1	J_2
M_1	**1**	**1**
M_2	**10**	**10**

$Q|\mathrm{pmtn}|\Sigma C_j$

- **Algorithm is SRPT-FM. Shortest Remaining Processing Time on the Fastest Machines.**

- **What about preemption in other models?**
- **P – doesn't help**
- **R – NP-complete**