Scheduling Notation

We will give the typical notation used in the course
Jobs

- Number: \( n \)
- Typical Index: \( j \)
- Features:
  - processing time: \( p_j \) or \( p_{ij} \)
  - release date: \( r_j \)
  - deadline or due date: \( d_j \)
  - weight \( w_j \)
Machines

– Number: \( m \)
– Typical Index: \( i \)
– Possible Environments:
  * 1 : one machine
  * \( P, P_m \): parallel (identical machines)
  * \( Q, Q_m \): related machines (different speeds)
  * \( R, R_m \): unrelated machines (processing time depends on job and machine)
– Shop Environments
  * \( J \): job shop – each job has linear constraints among its task
  * \( F \): flow shop – each job has the same linear constraints among its task
  * \( O \): open shop – no constraints among tasks
Constraints

We give some examples here:

- $r_j$: release date
- $\text{pmtn}$: preemption
- $\text{prec}$: precedence constraints
- $s_{jk}$: sequence dependent set up times
- $\text{bkdwn}$: machines may breakdown
- $\text{block}$: limited buffer size
Objectives

– A schedule designates which job runs on which machine at each time. It therefore assigns a completion time $C_j$ to each job $j$.
– We evaluate a job by some function of $C_j$ and the other parameters of job, e.g.
  * Lateness: $L_j = C_j - d_j$
  * Tardiness: $T_j = \max\{L_j, 0\}$
  * Unit Cost: $U_j = 1$ if $C_j > d_j$ and 0 otherwise
  * Flow (Response) Time: $F_j = C_j - r_j$
  * Idle Time: $I_j = C_j - r_j - p_j$
  * Stretch: $S_j = (C_j - r_j)/p_j$
– We then evaluate a schedule by some function of the job functions, usually a minimization of a
  * sum
  * weighted sum
  * discounted weighted sum
  * maximum (We use $X_{\max}$ as shorthand for $\max_j X_j$).
3 field notation

- machines — constraints — objective
- Default is no preemption

Examples:
- $P||C_{\text{max}}$ - parallel identical machines, minimize the schedule length (makespan)
- $1|\text{prec, pmtn}|\Sigma w_j C_j$ - one machine, precedence constraints and preemption, minimize the sum of weighted completion times
- $P\infty|\text{prec}|C_{\text{max}}$ - project scheduling
- $Jm|\text{nowait}|C_{\text{max}}$ - nowait job shop scheduling, minimize makespan
- $1|\text{pmtn}|\Sigma w_j T_j$ - one machine, preemption, minimum weighted tardiness