### **NP-complete Partitioning Problems**

Subset Sum: Given a list of t positive integers  $S = \{x_1, x_2, \dots, x_t\}$  and an integer B, is there a subset  $S' \subseteq S$  s.t.  $\sum_{x_i \in S'} x_i = B$ .

- Yes instance:  $S = \{1, 2, 5, 7, 8, 10, 11\}, B = 22$ .
- No instance:  $S = \{4, 10, 11, 12, 15\}, B = 28$ .

**3-Partition** Given a list of 3t positive integers  $S = \{x_1, x_2, \ldots, x_{3t}\}$  with  $\sum_{x_i \in S} x_i = tB$ , and each  $x_i$  satisfying  $B/4 < x_i < B/2$ , can you partition S into t groups of size 3, such that each group sums to exactly B.

- Yes instance:  $S = \{26, 26, 27, 28, 29, 29, 31, 33, 39, 40, 45, 47\}$
- No instance:  $S = \{26, 26, 27, 28, 29, 29, 31, 33, 38, 40, 45, 48\}$  (I think)

## $P||C_{\max}|$

Problem: Given n jobs with processing times  $p_j$ , schedule them on m machines so as to minimize the makespan.

Decision version: Given n jobs with processing times  $p_j$  and a number D, can you schedule them on m machines so as to complete by time D.

### Sample inputs:

- Jobs are  $\{1, 2, 5, 7, 8, 10, 11\}$ , 2 machines, D = 22.
- Jobs are  $S = \{4, 10, 11, 12, 15\}$ , 3 machines D = 20.

**Reduction:** Subset sum reduces to  $P||C_{\max}$ .

Idea of reduction: Given a subset sum instance, create a 2-machine instance of  $P||C_{\max}$ , with  $p_j = x_j$  and D = B. Now there is a feasible schedule iff there is a subset summing to B.

# $1|r_j|L_{\max}$

**Reduction:** Reduce 3-partition to  $1|r_j|L_{\text{max}}$ .

**3-Partition** Given a list of 3t positive integers  $S = \{x_1, x_2, \ldots, x_{3t}\}$  with  $\sum_{x_i \in S} x_i = tB$ , can you partition S into t groups of size 3, such that each group sums to exactly B.

Given a 3-partition instance, we will creat a  $1|r_j|L_{\text{max}}$  instance in the following way:

Jobs: n = 4t - 1 jobs, t - 1 of which are dummy jobs

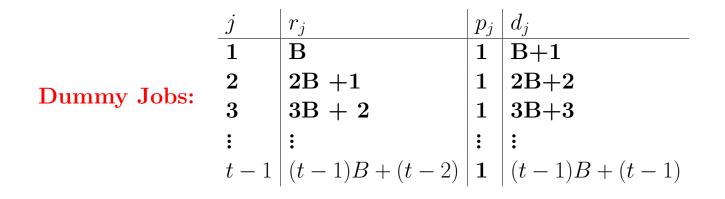
Dummy Jobs:	j	$\mid r_{j}$	$p_{j}$	$\mid d_{j}$
	1	В	1	B+1
	2	2B + 1	1	2B+2
	3	$3\mathrm{B}+2$	1	3B+3
	:	•	:	:
	t-1	(t-1)B + (t-2)	1	(t-1)B + (t-1)

### **Real Jobs:**

- indexed t through 4t-1.
- All have  $r_j = 0$

- All have  $d_j = tb + (t-1)$
- $p_j = x_{j-(t-1)}$

## Proof



### **Real Jobs:**

- indexed t through 4t-1.
- All have  $r_j = 0$
- All have  $d_j = tb + (t 1)$
- $p_j = x_{j-(t-1)}$

Idea of Proof: Argue that there is a schedule with  $L_{\text{max}} = 0$  iff the partition instance is yes.