IEOR 4405

Production Scheduling

Practice Problems for Midterm

1. [20 POINTS total] For each of the following scheduling problems, either say that it is NP-hard, or give the name of the algorithm which solves the problem optimally. No further explanation is required.

a) $1||\sum w_j C_j$
b) $1|r_j|\sum C_j$
c) $1|r_j, pmtn|L_{\max}$
d) $P|prec|C_{\max}$
e) $P||C_{\max}$
f) $R||\sum C_j$
g) $Q|pmtn|\sum C_j$

Question 2[15 POINTS] This question deals with lower bounds. Suppose we have 4 jobs with processing times $60, 50, 10, 1$. Explain what lower bound we can compute for each of the following problems. Restrict yourself to "easily computed" lower bound (e.g. do not solve a linear program or solve some other scheduling problem):

a) $P2||C_{\max}$
b) $Q|pmtn|C_{\max}$, assuming that the machines have speeds 6, 4, and 1.
c) $P2|prec|C_{\max}$, assume that the precedence constraints are of the form $60 \prec 10$, $50 \prec 10$, and $10 \prec 1$.

Question 3[15 POINTS]

The problem $1||\sum w_j U_j$ is an NP-hard problem. However, given enough additional information, it is possible to efficiently find the optimal solution.

a) Suppose that someone told you which jobs complete by their deadlines and which do not. Explain how to use this information to output an optimal solution.

b) Suppose that someone told you that k jobs complete by their deadlines and revealed to you $k-1$ of the jobs. Explain how to use this information to output an optimal solution.

Question 4 [15 POINTS] Suppose that you wanted to solve the problem $1||\sum 2^{C_j}$. (This is a new metric, not one studied in class.) Explain what algorithm you would use, and prove that it is optimal (Hint: an exchange argument will be helpful)

Question 5[10 POINTS] In the 4-partition problem, you are given $4t$ non-negative integers and a number b and you wish to know whether you can split the numbers into t groups of four numbers, with each group summing to exactly b. (In this problem there are no further restrictions on the range of the values.) Prove that this problem is NP-hard, by giving a reduction from 3-partition.