Stochastic Scheduling

Models Real World Uncertainty

- processing times
- arrivals
- machine availability
- ...

Our Model:

- Distribution over job data known in advance.
- Realization only known when job arrives/completes or when it can be inferred.

Example:

\[ p_j = \begin{cases} 
1 & \text{Pr} = 1/2 \\
3 & \text{Pr} = 1/2 
\end{cases} \]

After 1 unit of time, if the job doesn’t complete, we know that it will take 3 units.
Example

\[ p_1 = \begin{cases} 
1 & \Pr = 1/2 \\
9 & \Pr = 1/2 
\end{cases} \]

\[ p_2 = \begin{cases} 
4 & \Pr = 1/4 \\
6 & \Pr = 1/2 \\
8 & \Pr = 1/4 
\end{cases} \]

**Problem:** \( 1||\sum C_j \)

**Question:** What is the right algorithm? Is there still a simple ordering rule
Comparing random variables

- **Density Function:** \( f(x) \)
- **Distribution Function:** \( F(x) = P(X \leq t) = \int_0^t f(x) \, dx \)

**Definitions of \( X_1 \geq X_2 \)**

- **Larger in Expectation:** \( E(X_1) \geq E(X_2) \)
- **Stochastically larger:** \( \forall t : P(X_1 > t) \geq P(X_2 > t) \)
- **Almost surely larger:** \( P(X_1 \geq X_2) = 1 \)
Another example, $P||C_{\text{max}}$

Case 1: $p_1 = p_2 = 1$

Case 2: $p_1 = 1$

\[
p_2 = \begin{cases} 
0 & \text{Pr} = 1/2 \\
2 & \text{Pr} = 1/2 
\end{cases}
\]

Case 3:

\[
p_1 = p_2 = \begin{cases} 
0 & \text{Pr} = 1/2 \\
2 & \text{Pr} = 1/2 
\end{cases}
\]

Case 4: $p_1, p_2$ both uniform in $[0, 2]$. 
Objective Values

1. $C_{\text{max}} = 1$
2. $C_{\text{max}} = 3/2$
3. $C_{\text{max}} = 3/2$
4. $C_{\text{max}} = 4/3$
Different Models of Stochastic Scheduling

Models of Knowledge

- static: Choose order of jobs based on distribution only
- dynamic: Choose order of jobs based on knowledge gained when running
  Also consider Preemption vs. Non-preemption

Example:

- $1||\sum U_j$
- 3 jobs with same distribution:

$$p_j = \begin{cases} 
2 & \Pr = 1/2 \\
8 & \Pr = 1/2 
\end{cases}$$

$$d_j = \begin{cases} 
1 & \Pr = 1/2 \\
5 & \Pr = 1/2 
\end{cases}$$

What is the expected objective value for:

- static non-preemptive
- dynamic non-preemptive
- dynamic preemptive
Another Example

- **Problem:** $1|\text{pmtn}| \sum C_j$

- **Jobs:**

  \begin{align*}
  p_1 &= \begin{cases} 
  1 & \Pr = 1/2 \\
  3 & \Pr = 1/2
  \end{cases} \\
  p_2 &= \begin{cases} 
  2 & \Pr = 1/2 \\
  4 & \Pr = 1/2
  \end{cases} \\
  p_3 &= \begin{cases} 
  1 & \Pr = 1/2 \\
  7 & \Pr = 1/2
  \end{cases}
  \end{align*}