Tanker Scheduling

Ships have:
- capacity
- draught (minimum depth to float)
- range of speeds and fuel consumption
- location and available time

Ports have:
- weight limits
- draught
- other physical restrictions
- government restrictions
Cargo has

- type
- load port
- destination port
- time constraints
- load and unload times

A company will own ships and may rent ships. It is more expensive to rent ships.

Objective: minimize cost

- operating costs for company ships
- charter rates
- fuel costs
- port charges
Formulation

Notation:
Parameters

- $n$ - number of cargoes
- $T$ - number of company owned tankers
- $p$ - number of ports

plus data for all of the above.

Compute

- $S_i$ - the set of possible schedules for ship $i$. $a_{ij}^l = 1$ if under schedule $l$ ship $i$ transports cargo $j$.
- $c_j^*$ is amount paid to transport cargo $j$ on a ship that is not company owned.
- $c_i^l$ - incremental cost of operating a company-owned ship $i$ under schedule $l$ versus keeping ship $i$ idle.

Compute the profit for operationg ship $i$ according to schedule $l$ as $\pi_i^l = \sum_{j=1}^n a_{ij}^l c_j^* - c_i^l$. 
Decision variable:  
\[ x_i^l \] if ship \( i \) follows schedule \( l \).

Formulation

\[
\text{maximize} \quad \sum_{i=1}^{T} \sum_{l \in S_i} \pi_i^l x_i^l \\
\text{subject to} \\
\sum_{i=1}^{T} \sum_{l \in S_i} a_{ij}^l x_i^l \leq 1 \quad j = 1, \ldots, n \\
\sum_{l \in S_i} x_i^l \leq 1 \quad i = 1, \ldots, T \\
x_i^l \in \{0, 1\} \quad l \in S_i, i = 1, \ldots, T
\]

Solution  Set packing. Use branch and bound.
Example

• 3 ships
• 12 cargoes

Analysis of the data show that each of the ships has five feasible schedules:

| Schedules | \( a_{1j} \) | \( a_{2j} \) | \( a_{3j} \) | \( a_{4j} \) | \( a_{5j} \) | \( a_{1j} \) | \( a_{2j} \) | \( a_{3j} \) | \( a_{4j} \) | \( a_{5j} \) | \( a_{1j} \) | \( a_{2j} \) | \( a_{3j} \) | \( a_{4j} \) | \( a_{5j} \) |
|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| cargo 1   | 1           | 0           | 0           | 1           | 1           | 0           | 1           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1           | 0           |
| cargo 2   | 1           | 0           | 0           | 0           | 0           | 1           | 0           | 0           | 0           | 0           | 0           | 1           | 0           | 1           | 1           |             |
| cargo 3   | 0           | 0           | 1           | 0           | 1           | 0           | 0           | 0           | 1           | 1           | 0           | 0           | 0           | 0           | 0           |             |
| cargo 4   | 0           | 1           | 1           | 1           | 0           | 1           | 0           | 1           | 0           | 0           | 0           | 0           | 0           | 0           | 0           |             |
| cargo 5   | 1           | 1           | 0           | 0           | 0           | 0           | 0           | 0           | 1           | 0           | 0           | 0           | 1           | 0           | 1           |             |
| cargo 6   | 0           | 0           | 0           | 1           | 1           | 0           | 1           | 0           | 0           | 1           | 1           | 0           | 0           | 0           | 0           |             |
| cargo 7   | 0           | 0           | 0           | 0           | 0           | 0           | 0           | 1           | 1           | 0           | 0           | 0           | 0           | 0           | 0           | 1           |
| cargo 8   | 0           | 1           | 0           | 0           | 0           | 0           | 1           | 0           | 1           | 1           | 1           | 0           | 0           | 0           | 0           |             |
| cargo 9   | 0           | 0           | 1           | 0           | 0           | 0           | 1           | 0           | 0           | 1           | 1           | 1           | 1           | 1           | 0           |             |
| cargo 10  | 0           | 1           | 0           | 0           | 0           | 1           | 0           | 0           | 0           | 0           | 1           | 1           | 0           | 0           | 0           |             |
| cargo 11  | 0           | 0           | 0           | 0           | 0           | 0           | 1           | 1           | 0           | 0           | 0           | 1           | 1           | 1           | 0           |             |
| cargo 12  | 0           | 0           | 0           | 1           | 0           | 0           | 0           | 0           | 0           | 1           | 0           | 1           | 1           | 1           | 1           |             |
Costs

Charter cost (CC) for transporting a particular cargo by charter:

<table>
<thead>
<tr>
<th>Cargo</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>1429</td>
<td>1323</td>
<td>1208</td>
<td>512</td>
<td>2173</td>
<td>2217</td>
<td>1775</td>
<td>1885</td>
<td>2468</td>
<td>1928</td>
<td>1634</td>
<td>741</td>
</tr>
</tbody>
</table>

Operating costs of the tankers under each one of the schedules is also given:

<table>
<thead>
<tr>
<th>Schedule</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost of tanker 1 ($c_1$)</td>
<td>5608</td>
<td>5033</td>
<td>2722</td>
<td>3505</td>
<td>3996</td>
</tr>
<tr>
<td>cost of tanker 2 ($c_2$)</td>
<td>4019</td>
<td>6914</td>
<td>4693</td>
<td>7910</td>
<td>6866</td>
</tr>
<tr>
<td>cost of tanker 3 ($c_3$)</td>
<td>5829</td>
<td>5588</td>
<td>82824</td>
<td>3338</td>
<td>4715</td>
</tr>
</tbody>
</table>

We can compute the profit for each schedule

<table>
<thead>
<tr>
<th>Schedule</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>profit of tanker 1 ($\pi_1$)</td>
<td>-683</td>
<td>1465</td>
<td>1466</td>
<td>1394</td>
<td>858</td>
</tr>
<tr>
<td>profit of tanker 2 ($\pi_2$)</td>
<td>1629</td>
<td>834</td>
<td>1113</td>
<td>-869</td>
<td>910</td>
</tr>
<tr>
<td>profit of tanker 3 ($\pi_3$)</td>
<td>1525</td>
<td>1765</td>
<td>-1268</td>
<td>1789</td>
<td>1297</td>
</tr>
</tbody>
</table>
Now we can give an IP

\[
\text{maximize} \quad -733x_1^1 + 1465x_1^2 + 1466x_1^3 + 1394x_1^4 + 858x_1^5 \\
+1629x_2^1 + 834x_2^2 + 1113x_2^3 + -869x_2^4 + 910x_2^5 \\
+1525x_3^1 + 1765x_3^2 + -1268x_3^3 + 1789x_3^4 + 1297x_3^5
\]

subject to
\[
\begin{align*}
x_1^1 + x_1^4 + x_1^5 + x_2^1 + x_3^1 & \leq 1 \\
x_1^1 + x_2^2 + x_3^2 + x_3^4 + x_3^5 & \leq 1 \\
x_1^3 + x_1^5 + x_2^4 + x_2^5 & \leq 1 \\
x_1^2 + x_1^3 + x_1^4 + x_1^2 + x_2^3 & \leq 1 \\
x_1^1 + x_1^2 + x_1^4 + x_3^3 + x_3^5 & \leq 1 \\
x_1^4 + x_1^1 + x_2^2 + x_2^4 + x_3^1 & \leq 1 \\
x_1^3 + x_2^4 + x_2^5 & \leq 1 \\
x_1^2 + x_2^1 + x_2^3 + x_2^4 + x_2^5 & \leq 1 \\
x_1^3 + x_2^2 + x_2^5 + x_3^1 + x_3^2 + x_3^3 & \leq 1 \\
x_1^2 + x_2^1 + x_3^1 + x_3^2 & \leq 1 \\
x_2^2 + x_3^3 + x_3^2 + x_3^4 & \leq 1 \\
x_1^4 + x_3^1 + x_3^3 + x_3^4 + x_3^5 & \leq 1 \\
x_1^1 + x_1^2 + x_3^1 + x_3^4 + x_3^5 & \leq 1 \\
x_2^1 + x_2^2 + x_3^3 + x_3^4 + x_5^5 & \leq 1 \\
x_1^1 + x_1^1 + x_1^3 + x_1^4 + x_1^5 & \leq 1 \\
x_2^1 + x_2^2 + x_2^3 + x_3^4 + x_5^5 & \leq 1
\end{align*}
\]
Optimal solution  Schedule 3 for ship 1, schedule 4 for ship 3. Ship 2
remains idle. Cargoes 5,6,7,8,10 are transported by charters. Value =
3255.
Train timetabling

- One track with many stations (think 1/9 subway line or commuter rail).
- Trains can pass at stations but not between stations.
- Stations are numbered 0 to L.
- Tracks are numbered 1 to $L + 1$.
- Track $i$ connects station $j - 1$ with $j$.
- Time is measured in minutes (1 to 1440).
- You are given preferred arrival, departure, travel, and holdover times for each train at each station.
- There is a piecewise linear function measuring the cost (lost revenue) of deviating from the desired time.
The desired arrival time is 800. The graph shows the cost as a function of time, with a minimum cost between 780 and 800.
IP

Variables

• \( y_{ij} = \) time train \( i \) enters link \( j \) (leaves station \( j - 1 \))
• \( z_{ij} = \) time train \( i \) exits line \( j \) (arrives at station \( j \))

We compute

• \( \tau_{ij} = z_{ij} - y_{ij} \) (travel time of train \( i \) in link \( j \))
• \( \delta_{ij} = y_{i,j+1} - z_{ij} \) (dwelling time of train \( i \) in station \( j \))

We are given costs for each of these quantities:

• \( c_{ij}^a(z_{ij}) \) - costs for train \( i \) arriving at station \( j \)
• \( c_{ij}^d(y_{ij}) \) - costs for train \( i \) departing from station \( j \)
• \( c_{ij}^\tau(\tau_{ij}) \) - costs for travel time of train \( i \) in link \( j \)
• \( c_{ij}^\delta(\delta_{ij}) \) - costs for travel time of train \( i \) dwelling in station \( j \).

Each of these costs is piecewise linear, and we are given min and max values. Also, minimum and maximum headway values \( H \)

\[ T \] is the set of possible trains.

Variable: \( x_{hij} = 1 \) is train \( h \) immediately precedes train \( i \) on link \( j \).
\begin{align*}
\text{minimize} & \quad \sum_{i \in T} \sum_{j=1}^{L} \left( c_{ij}^a(z_{ij}) + c_{ij-1}^d(y_{ij}) + c_{ij}^\tau(\tau_{ij}) \right) + \sum_{i \in T} \sum_{j=1}^{L-1} \left( c_{ij}^\delta(\delta_{ij}) \right) \\
\text{subject to} & \\
& \quad y_{ij} \geq y_{ij}^\text{min} \quad i \in T, j = 1, \ldots, L \\
& \quad y_{ij} \leq y_{ij}^\text{max} \quad i \in T, j = 1, \ldots, L \\
& \quad z_{ij} \geq z_{ij}^\text{min} \quad i \in T, j = 1, \ldots, L \\
& \quad z_{ij} \leq z_{ij}^\text{max} \quad i \in T, j = 1, \ldots, L \\
& \quad \tau_{ij} = z_{ij} - y_{ij} \quad i \in T, j = 1, \ldots, L \\
& \quad \tau_{ij} \geq \tau_{ij}^\text{min} \quad i \in T, j = 1, \ldots, L \\
& \quad \tau_{ij} \leq \tau_{ij}^\text{max} \quad i \in T, j = 1, \ldots, L \\
& \quad \delta_{ij} = y_{i,j+1} - z_{ij} \quad i \in T, j = 1, \ldots, L \\
& \quad \delta_{ij} \geq \delta_{ij}^\text{min} \quad i \in T, j = 1, \ldots, L - 1 \\
& \quad \delta_{ij} \leq \delta_{ij}^\text{max} \quad i \in T, j = 1, \ldots, L - 1 \\
& \quad y_{i,j+1} - y_{h,j+1} + (1 - x_{hij})M \geq H_{hij}^d \quad i \in T, j = 1, \ldots, L \\
& \quad z_{ij} - z_{hj} + (1 - x_{hij})M \geq H_{hij}^a \quad i \in T, j = 1, \ldots, L \\
& \quad \sum_{h \in \{T-i\}} x_{hij} = 1 \quad i \in T, j = 1, \ldots, L \\
& \quad x_{hij} \in \{0, 1\} 
\end{align*}
Solution

Can solve using heuristic similar to shifting bottleneck heuristic. Set one train (by importance) and resolve LP relaxation.