Basics of Algorithm Analysis

Problem vs. Instance vs. Algorithm vs. Solution

- **Problem : minimize makespan on 2 machines** $P||C_{\max}$
- **Instance:** 5 **jobs with processing times** $(4, 1, 8, 5, 6)$
- **Algorithm: Alternate putting the jobs on machine** 1 **and machines** 2**.**
- **Solution: Machine** 1 **has jobs** J_1, J_3, J_5 **with total processing time 18, machine** 2 **has jobs** J_2, J_4 **with total processing time 6. Makespan is 18.**

Goals:

- **We want to develop algorithms that, on "any" instance, will produce good solutions.**
- **We want to understand how our algorithms perform, so that, given a new instance, we can predict how long they will take and what kind of solution they return.**

Basics of Algorithm Analysis

Running Time: Given an algorithm, and an input of size n, we wish to know the running time as a function of n.

- We measure running time as a function of n, the size of the input (in bytes assuming a reasonable encoding).
- We work in the RAM model of computation. All "reasonable" operations take "1" unit of time. (e.g. +, *, -, /, array access, pointer following, writing a value, one byte of I/O...)

What is the running time of an algorithm

- Best case (seldom used)
- Average case (used if we understand the average)
- Worst case (used most often)

We measure as a function of n, and ignore low order terms.

- $5n^3 + n - 6$ becomes n^3
- $8n \log n - 60n$ becomes $n \log n$
- $2^n + 3n^4$ becomes 2^n

Asymptotic notation

big-O

$$O(g(n)) = \{f(n) : \textbf{ there exist positive constants } c \textbf{ and } n_0 \textbf{ such that } 0 \le f(n) \le cg(n) \textbf{ for all } n \ge n_0\} \ .$$

Alternatively, we say

$$f(n) = O(g(n)) \textbf{ if there exist positive constants } c \textbf{ and } n_0 \textbf{ such that } 0 \le f(n) \le cg(n) \textbf{ for all } n \ge n_0\}$$

Informally, $f(n) = O(g(n))$ **means that** $f(n)$ **is asymptotically less than or equal to** $g(n)$**.**

Classification **We use these to classify algorithms into classes, e.g.** n, n^2, $n \log n$, 2^n.

Simple Rules

- **Nested loops multiply (even when the inner loop is from 1 to the outer loop value).**
- **Sequential loops add**
- **Repeated halving is linear.**

Examples

FindMax (A, n)

```
// A is an array of length n
maxval = A[1]
for i = 2 to n
        if (A[i] > maxval)
                maxval = A[i]
return maxval
```

Running time is $O(n)$

Examples

MatMult (A, B, C, p, q, r)

```
// A is p × q; B is q × r; C is p × r
for i = 1 to p
    for j = 1 to r
        C[i, j] = 0
        for k = 1 to q
            C[i, j] = C[i, j] + A[i, k] * B[k, j]
```

Running Time is $O(pqr)$ **. If matrices are** $n \times n$**, then** $O(n^3)$ **.**

Some Things to Know

- Sorting n numbers takes $O(n \log n)$ time.
- Finding a shortest path with non-negative weights in a graph with n nodes and m edges takes $O(m \log n)$ time.
- You can use a Priority Queue Data Structure to maintain a set of n numbers, and insert and delete the maximum in $O(\log n)$ time per operation.