

# IEOR 4405

## Production Scheduling

### Practice Final Exam

**Question 1 [10 POINTS total]**

For each of the following algorithms, give one scheduling problem for which the algorithm is optimal.

- a) SPT
- b) EDD
- c) SPT(1)-LPT(2)
- d) WDSEPT

**Question 2 [20 POINTS]** All parts of this question refer to the flow shop problem. Consider the following instance, in which entry  $(j, i)$  is the processing time of job  $j$  on machine  $i$ :

	$M_1$	$M_2$	$M_3$
$J_1$	4	8	1
$J_2$	3	5	4
$J_3$	10	1	6

The order for the machines is  $M_1, M_2, M_3$ .

Answer parts a through d by drawing a Gantt chart.

- a) Give an example of a permutation schedule. What is the makespan?
- b) Give an example of a schedule that is not a permutation schedule. What is the makespan?
- c) Show the schedule corresponding to the permutation (1,2,3) for a blocking flow shop. What is the makespan?
- d) Show the schedule corresponding to the permutation (1,2,3) for a no-wait flow shop. What is the makespan?
- e) If there is an appropriate graph for which the critical path will determine the makespan for any of parts a,b,c or d, please draw it. (For each part, there may or may not be an appropriate graph.)

**Question 3[10 POINTS]**

Give an integer program for  $1|prec, p_j = 1|L_{\max}$  in which variable  $x_{jt}$  is 1 if job  $j$  finishes at time  $t$ .

**Question 4 [15 POINTS total]**

Consider a scheduling instance of  $P||C_{\max}$  with the following 3 jobs:  
 job 1: deterministic processing time of 2  
 job 2: processing time of 1 with probability 1/2, and a processing time of 3 with probability

of  $1/2$

job 3: processing time is drawn uniformly from the interval  $[0, 2]$ .

a) Suppose that there is one machine. What is the expected makespan of the **optimal** schedule?

b) Suppose that there are three machines. What is the expected makespan of the **optimal** schedule?

c) Suppose that there are two machines. What is the expected makespan of the **optimal** schedule?

**Question 5[10 POINTS]** Consider the following problem. You have 2 jobs with processing times given from the distributions below. ( $X_1$  is the random variable for the processing time of job 1 and  $X_2$  is the random variable for the processing time of job 2.) You are allowed to preempt the machines at discrete times  $0, 1, 2, \dots$ . If job  $j$  is completed at time  $C_j$ , a reward  $w_j(.5)^{C_j}$  is received. You wish to maximize the total expected reward. Which job should run first in an optimal schedule? Should it ever be preempted? Why or why not? (Show your work for credit.)

$$\Pr\{X_1 = 1\} = 3/4 \quad \Pr\{X_1 = 2\} = 1/8 \quad \Pr\{X_1 = 3\} = 1/8$$

$$\Pr\{X_2 = 1\} = 1/2 \quad \Pr\{X_2 = 2\} = 1/2$$

$$w_1 = 10, w_2 = 5.$$

**Question 6[15 POINTS]** This question concerns the LPT algorithm for the problem  $P2||C_{\max}$ .

a) Consider instances with 3 jobs. Either argue that LPT is always optimal in this case, or give an example which shows that it is not.

b) Consider instances with 4 jobs. Either argue that LPT is always optimal in this case, or give an example which shows that it is not.

c) Consider instances with 5 jobs. Either argue that LPT is always optimal in this case, or give an example which shows that it is not.

d) Consider instances with 6 jobs. Either argue that LPT is always optimal in this case, or give an example which shows that it is not.