Scheduling Notation

We will give the typical notation used in the course
Jobs

• Number: $n$
• Typical Index: $j$
• Features:
  – processing time: $p_j$ or $p_{ij}$
  – release date: $r_j$
  – deadline or due date: $d_j$
  – weight $w_j$
Machines

- Number: $m$
- Typical Index: $i$
- Possible Environments:
  * 1: one machine
  * P,Pm: parallel (identical machines)
  * Q,Qm: related machines (different speeds)
  * R,Rm: unrelated machines (processing time depends on job and machine)
- Shop Environments
  * J: job shop – each job has linear constraints among its task
  * F: flow shop – each job has the same linear constraints among its task
  * O: open shop – no constraints among tasks

job = build a car consists of many tasks on many machines
Constraints

We give some examples here:

- \( r_j \): release date
- \( \text{pmtn} \): preemption
- \( \text{prec} \): precedence constraints
- \( s_{jk} \): sequence dependent set up times
- \( \text{bkdwn} \): machines may breakdown
- \( \text{block} \): limited buffer size
Objectives

- A **schedule** designates which job runs on which machine at each time. It therefore assigns a completion time $C_j$ to each job $j$.
- We evaluate a job by some function of $C_j$ and the other parameters of job, e.g.
  - Lateness: $L_j = C_j - d_j$
  - Tardiness: $T_j = \max\{L_j, 0\}$
  - Unit Cost: $U_j = 1$ if $C_j > d_j$ and 0 otherwise.
  - Flow (Response) Time: $F_j = C_j - r_j$
  - Idle Time: $I_j = C_j - r_j - p_j$
  - Stretch: $S_j = (C_j - r_j)/p_j$
- We then evaluate a schedule by some function of the job functions, usually a minimization of a
  - sum
  - weighted sum
  - discounted weighted sum
  - maximum (We use $X_{\text{max}}$ as shorthand for $\max_j X_j$).
3 field notation

- machines — constraints — objective

- Default is no preemption Examples:
  - $P || C_{\text{max}}$ - parallel identical machines, minimize the schedule length (makespan)
  - $1|\text{prec, pmtn}| \sum w_j C_j$ - one machine, precedence constraints and preemption, minimize the sum of weighted completion times
  - $P \infty|\text{prec}| C_{\text{max}}$ - project scheduling
  - $J_m|\text{nowait}| C_{\text{max}}$ - nowait job shop scheduling, minimize makespan
  - $1|\text{pmtn}| \sum w_j T_j$ - one machine, preemption, minimum weighted tardiness

$$C_{\text{max}} = \max \{ C_j \}$$