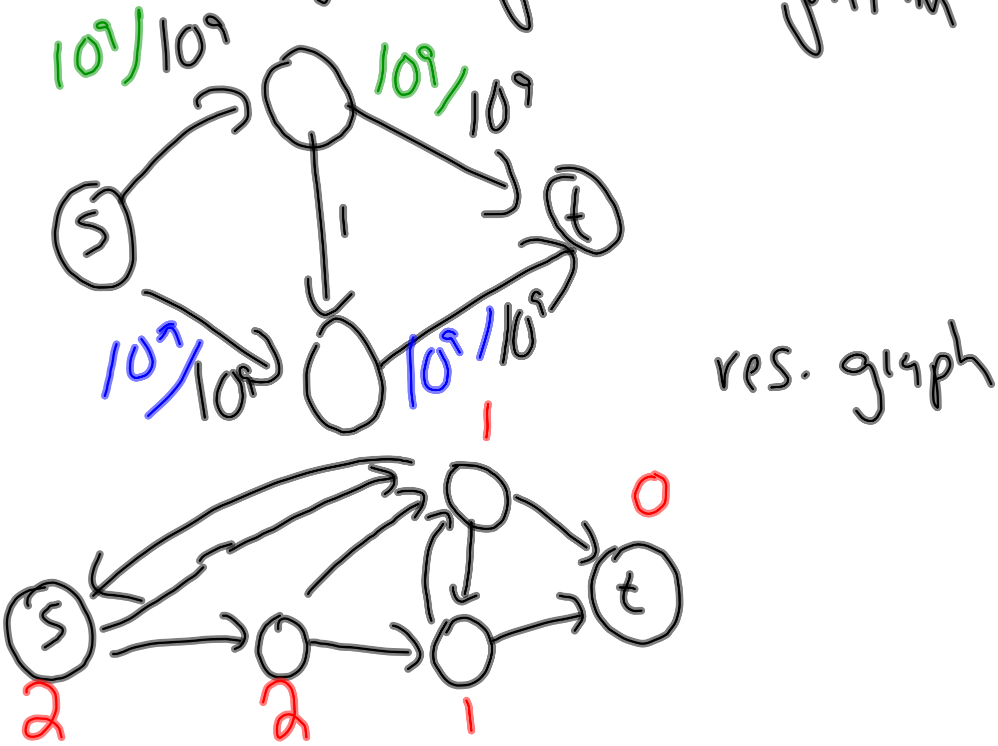
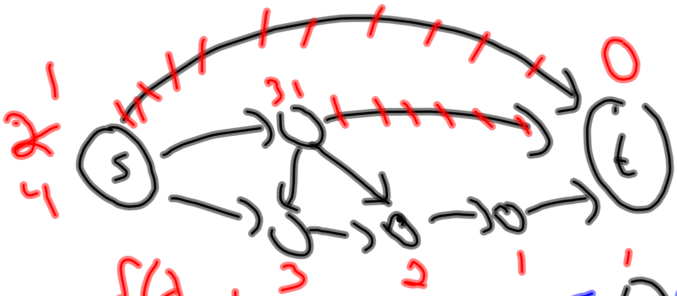
# Shortest Augmenting Path Algorithm



$\delta(v)$ distance from v to t



$\delta(s) = 1$

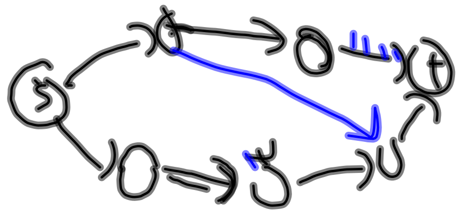
$\delta(s) = 2$

$\delta(s) = 2$

$\delta(s) = 4$

$\delta(s) = n-1$

$\delta(s) = \infty$



Feb 19-4:26 PM

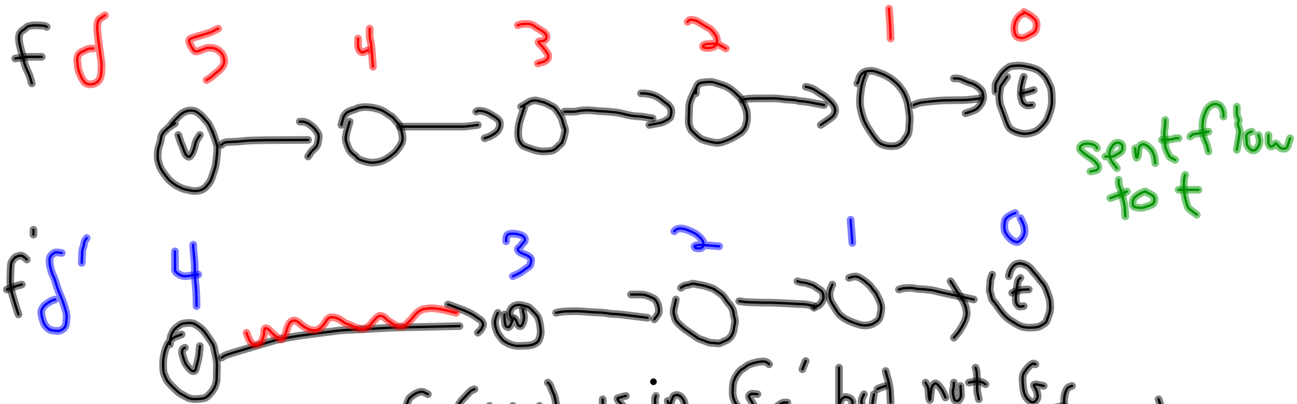
**Lemma** For every v, $\delta(v)$ never decreases

**Pf**

$\delta$ dist. before an aug. path

$\delta'$ dist. after

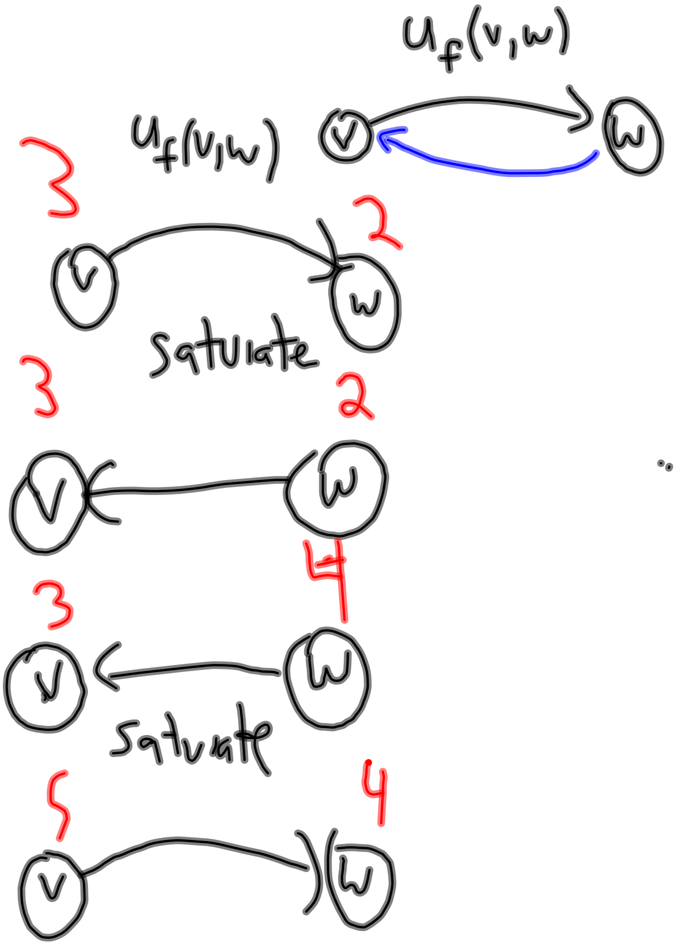
Suppose some $\delta(v)$ decreases, let v be the one w/ min $\delta(v)$ to decrease




If $(v,w)$ is in $G_{f'}$ but not $G_f$ then we sent flow on $(w,v)$ in the augmentation

$\Rightarrow \delta(w) = 6$, contradicts $\delta'(w) = 3$. contradiction

- $\delta(v)$ never decreases
- For each $v$ $\delta(v)$ increases $\le n$ times.
- Total # of times that any $\delta(v)$ increases $\le n^2$.




If a push sends $u_f(v,w)$ flow from $v$ to $w$, we call it saturating

## Facts

1) Between any 2 saturations of $(v,w)$ or $(w,v)$, either $\delta(v)$ or $\delta(w)$ must increase by 2.

\# saturations of $(v,w)$ or $(v,w)$ $\leq n$

2) Every aug. path saturates at least 1 residual edge.

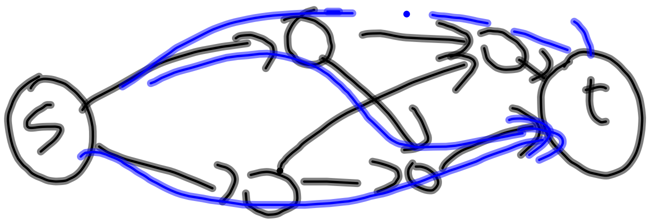
$$\# \text{aug paths} \leq \# \text{saturations} \leq n \cdot m = O(nm)$$

R.T is $O(nm^2)$

Edmonds, Karp 72

$O(n^2 m)$ a little careful

Blocking Flows. send along "~~many~~ all" shortest paths simultaneously

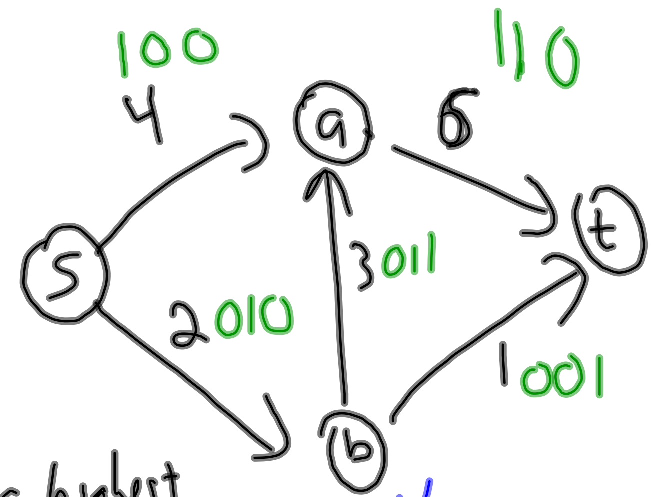



Repeat $n$ times

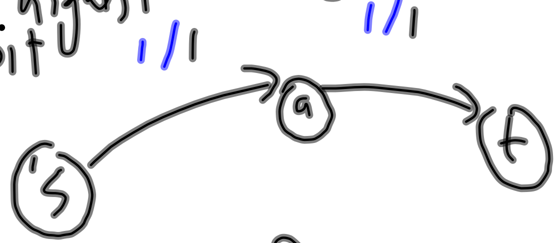
Data Structures

# Scaling of capacities

successive approximation

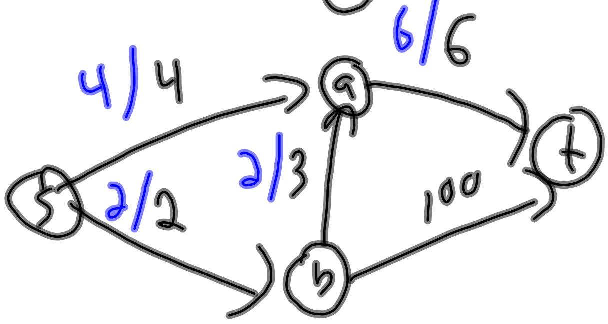



Consider highest order bit




Solve


update
add a bit to u
double the flows
→ doubles u
or doubles u +1



1. Correctly compute a max flow
   - always feasible
   - always sending flow on a residual path in $G_f$

2. Running time $\lg U$ iterations of scaling

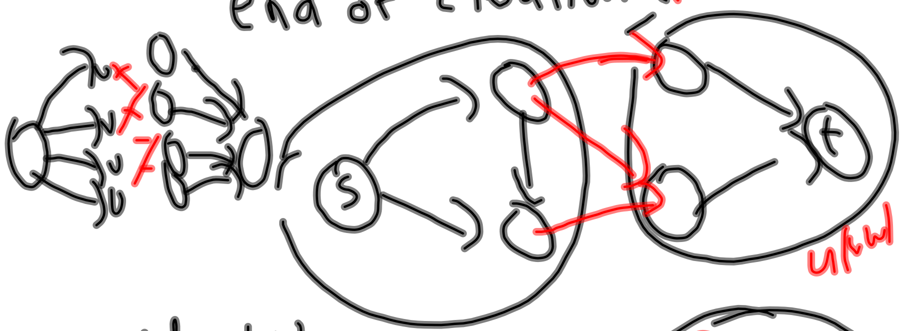
Each iteration is a sequence of x aug. paths

Bound how much flow gets sent in an iteration

end of iteration i

saturated edges $f = u$

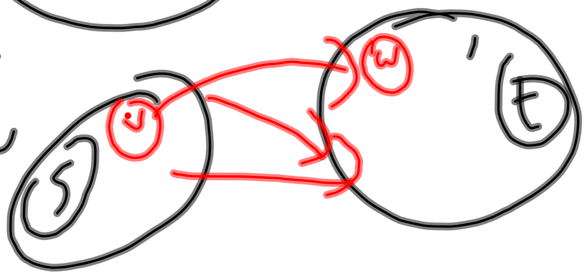
min cut



$u(vw) \le f(vw) \le u(vw)+1$

$u_f(vw) = 0$ or $1$

add a bit
double flow



$U_f(S,T) \le m$.

$\exists$ a cut in $G_f$ w/ $u_f(S,T) \leq m$

$\therefore$ total flow we can send in $G_f$ $\leq m$

$\therefore$ at most $m$ iterations of aug. paths.

$O(m^2 \lg U)$ time