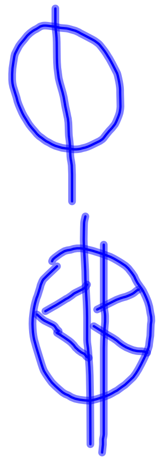
Carpool fairness

Person	Days 1	2	3	4	5
1	X	X	X		
2	X		X		
3	X	X	X	X	X
4		X	X	X	X

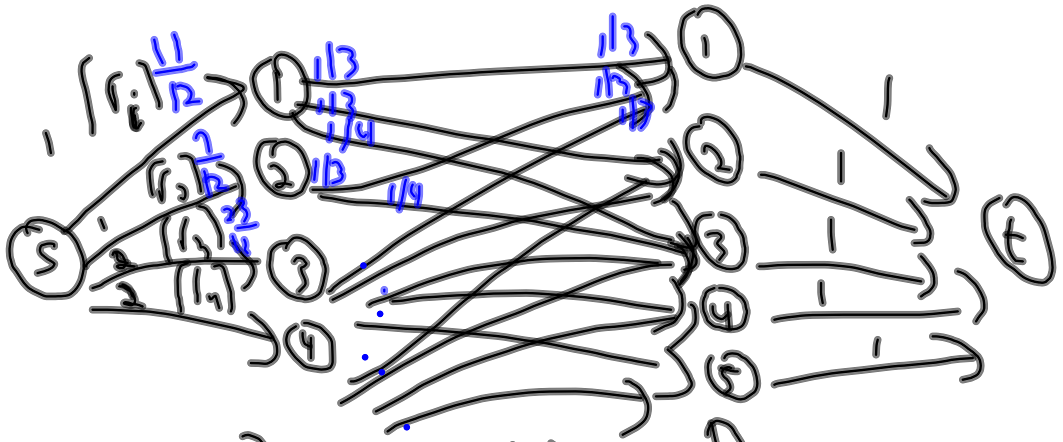


What's a fair division of driving?

Person	1	2	3	4	5	Responsibility
1	1/3	1/3	1/4			11/12
2	1/3		1/4	1/2	1/2	7/12
3	1/3	1/3	1/4	1/2	1/2	23/12
4		1/3	1/4			19/12

r_i $\quad \sum r_i = \#\text{days}$

Each person should drive no more than $\lceil r_i \rceil$ times.



Person $u(i,j) = 1$ Day

a flow "matches" person to a day

a flow of value 5 is a schedule for all 5 days

Does a flow of value 5 exist?

Use a fractional flow (values in table)

a fractional flow of value 5 exists,

$\therefore$ integral flow of value 5 exists.

Given a graph w/ int. caps.
$\exists$ an optimal integral flow.

$\Rightarrow$ If I have a fractional flow of value X
then there must exist an integral flow of
value $\lceil X \rceil$

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Baseball elimination (Sports writers end of Season problem)

Team	Wins w_i	Games left g_i	Against g_{ij}: NY	Bos	Tor	Bal
NY Yankees	93	8	–	1	6	1
Bos. Red Sox	89	4	1	–	0	3
Toronto Blue Jays	88	7	6	0	–	1
Baltimore Orioles	86	5	1	3	1	–

Which teams are eliminated, have no chance of winning?

Orioles are eliminated

Boston can win 93. Yankees lose all games

$\Rightarrow$ Toronto wins 6 games

$\Rightarrow$ Toronto wins 94

$$\frac{93+88+6}{2} = 93\tfrac{1}{2}$$

$$\min(w(NY), w(Tor)) \geq 94.$$

w_i = wins for team i

g_i = games left for i

g_{ij} : games left between i & j

For $R \subseteq T$ $(T = \{\text{teams}\})$

$$w(R) = \sum_{i \in R} w_i$$

$$g(R) = \sum_{i<j} g_{ij}$$

$$a(R) = \frac{w(R) + g(R)}{|R|}$$

If $i \in T$, $R \subseteq T - \{i\}$, and $a(R) > w_i + g_i$, then i is eliminated.

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k is not eliminated if

X_{ij} is # of wins i over j

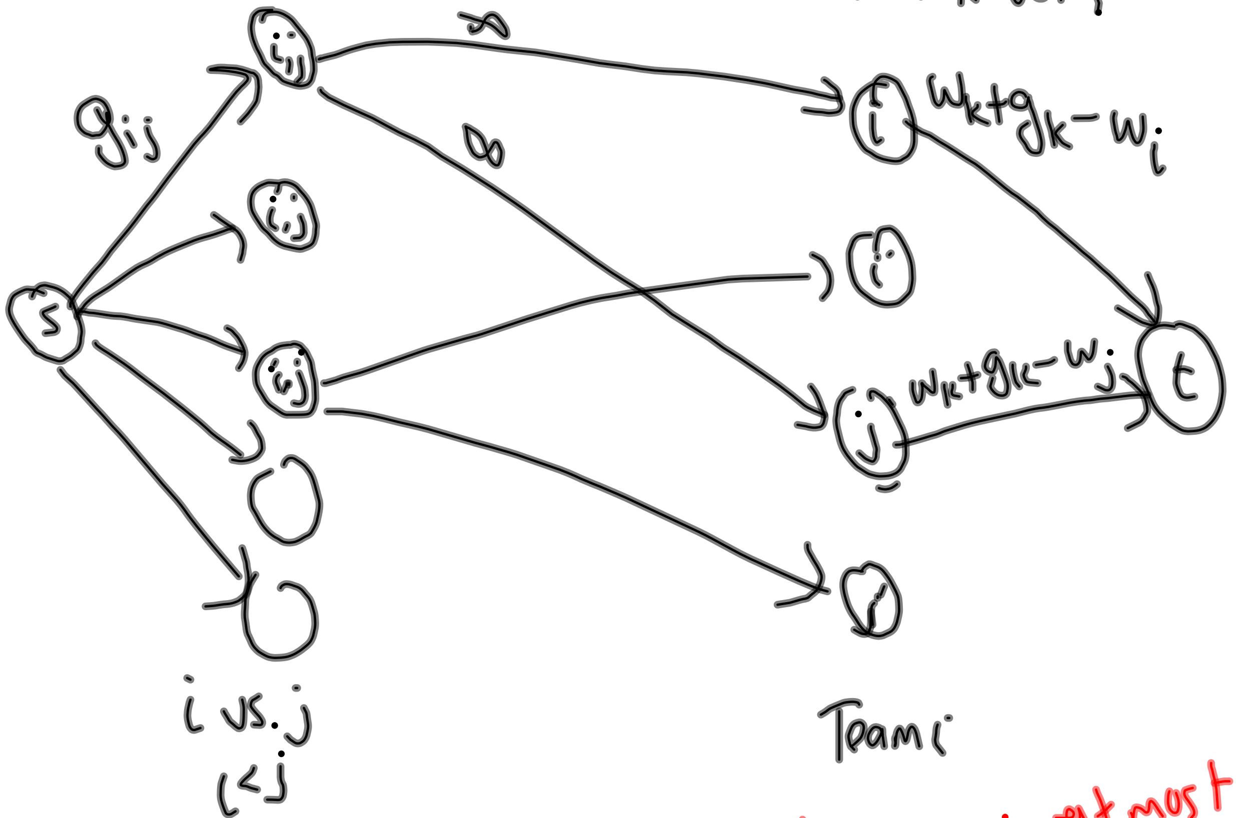
$\exists\, x_{ij}$

$$x_{ij} + x_{ji} = g_{ij} \quad \forall\, i,j \in T$$

$$w_k + \cancel{\sum_{j \in T} x_{kj}}^{\,g_k} \geq w_i + \sum_{j \in T - \{k\}} x_{ij} \quad \forall\, i \in T$$

$$x_{ij} \geq 0 \quad x_{ij} \text{ int}$$

some way to set the remaining games so that k comes out in first can k win



k can win $g_k + w_k$ games, can every other team win at most $g_k + w_k$ games

Claim

If a flow f of value $\sum_{i<j} g_{ij}$ exists, then k is not eliminated

Pf

f saturates all edges out of s, so each game is "set"

& f obeys capacities into sink, so each team i wins at most an additional $g_k + w_k - w_i$ games

$\therefore$ wins $\leq w_i + (g_k + w_k - w_i) = g_k + w_k$ games.

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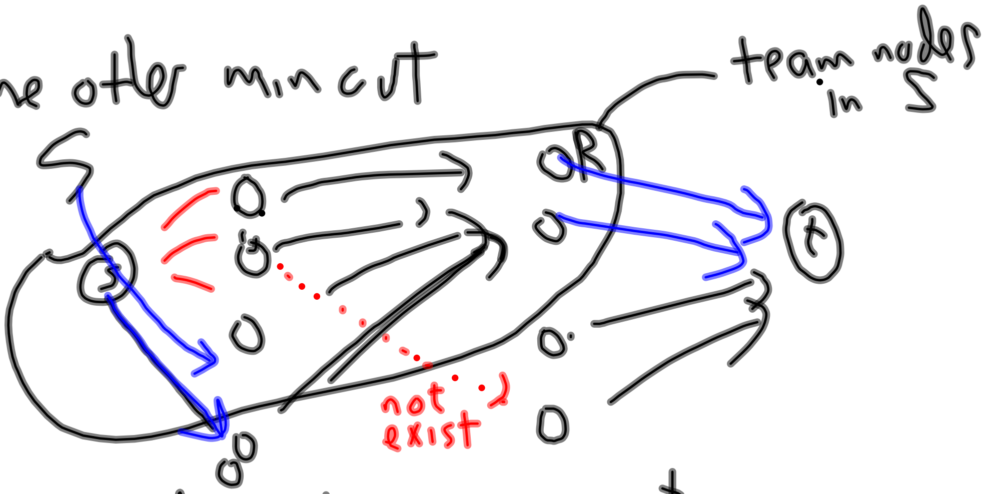
Claim

If no flow of value $\sum_{i<j} g_{ij}$ exists then k is eliminated.

Pf No flow of value $\sum_{i<j} g_{ij}$.

$\sum_{i<j} g_{ij} = u(s, V-\{s\})$ is not the min cut.

Some other min cut



no ∞ cap edges crossing min cut

$$u(S,\bar{S}) = \sum_{\{i,j\}\notin S} g_{ij} + \sum_{i\in R} w_k + g_k - w_i)$$

$$= \sum_{(i,j)\notin S} g_{ij} + |R|(w_k + g_k) - w(R)$$

$$\sum_{i<j} g_{ij} > \sum_{(i,j)\notin S} g_{ij} + |R|(w_k+g_k) - w(R)$$

non min-cut

$$g(R) \geq \sum_{(i,j)\in S} g_{ij} > |R|\,(w_k+g_k) - w(R).$$

if $(i,j)\in S$ then $i \,\&\, j \in R$

$$\frac{w(R)+g(R)}{|R|} > w_k + g_k \Rightarrow$$ k is elim.

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