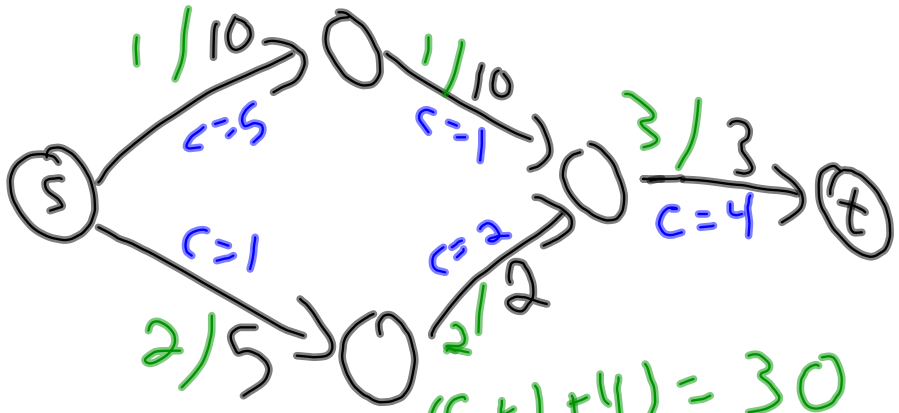
Min Cost Flows

1) Max flow of min cost.

C = cost
capacity



3 units on top: $3(5+1+4) = 30$

2 units on top
1 unit on bottom: $2(5+1+4)+1(1+2+4) = 27$

1 unit on top
2 .. on bottom: $1(5+1+4)+2(1+2+4) = 24$

2) Send x units from stat as cheaply as possible

x = 2, graph prev ex.

Send 2 units along bottom path

2(1+2+4) = 14

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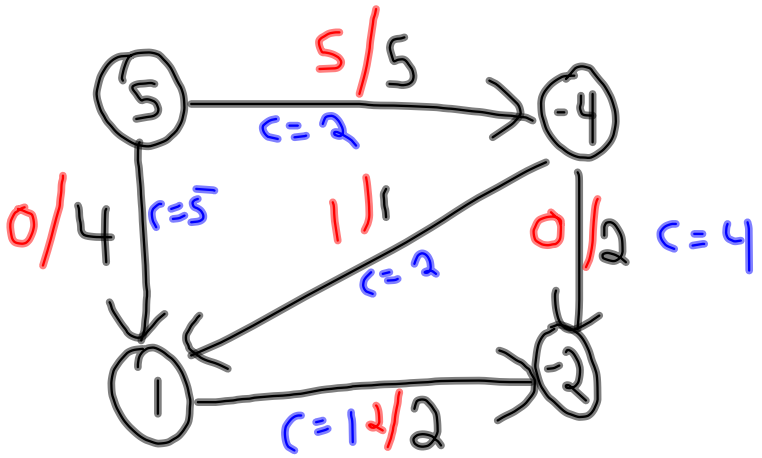
3) Supplies & demands



+ supply
− demand

Find flow sat. supp/dem of min cost.

$b(v)$ = s/d at node v

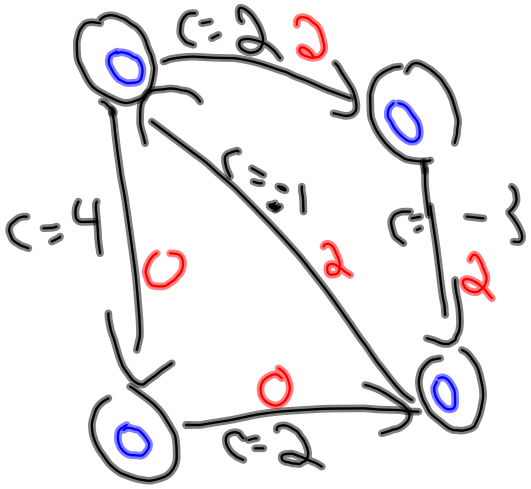
$\sum_v b(v) = 0$

$$\min \sum_{(v,w)\in E} c(v,w)\cdot f(v,w)$$

$$f(v,w) \le u(v,w) \quad \forall (v,w)\in E$$

$$\sum_{w\in V} f(v,w) - \sum_{w\in V} f(w,v) = b(v) \quad \forall v\in V$$

$$f(v,w) \ge 0$$



min-cost circulation
(all $b(v)=0$)

$4=2$ everywhere

$$\sum c(v,w)\cdot f(v,w)$$
$$= 2\cdot 2 + -3\cdot 2 + -1\cdot 2 =$$
$$-4$$