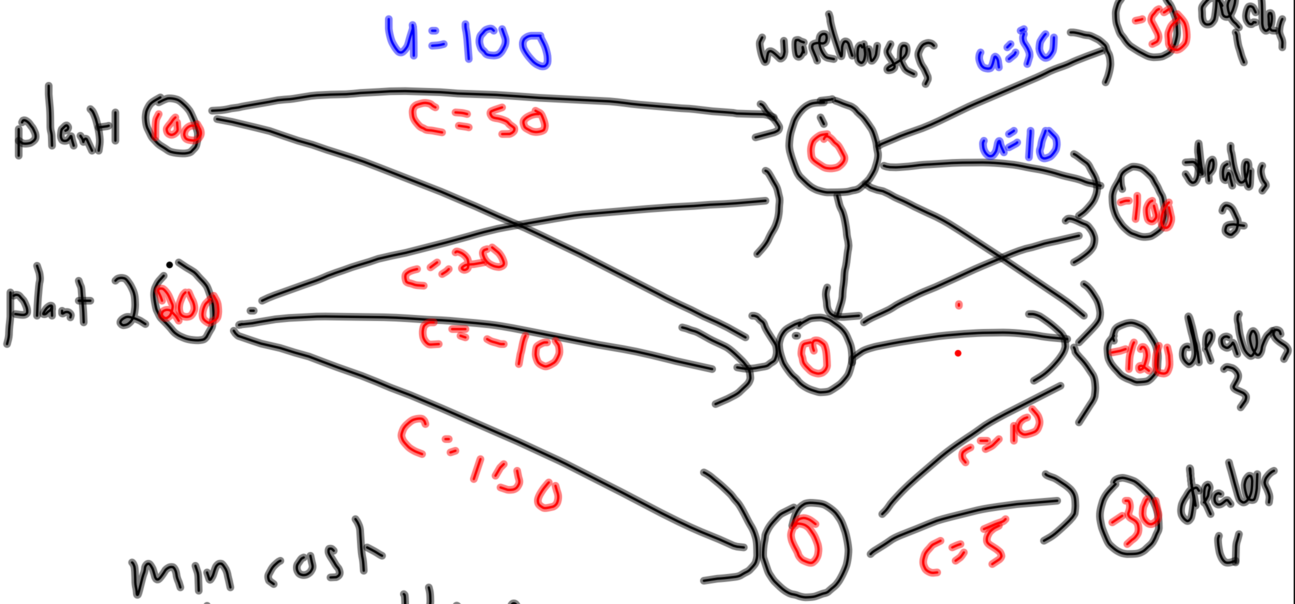
distribution network
GM



min cost
flow problem

Plane

holds p people

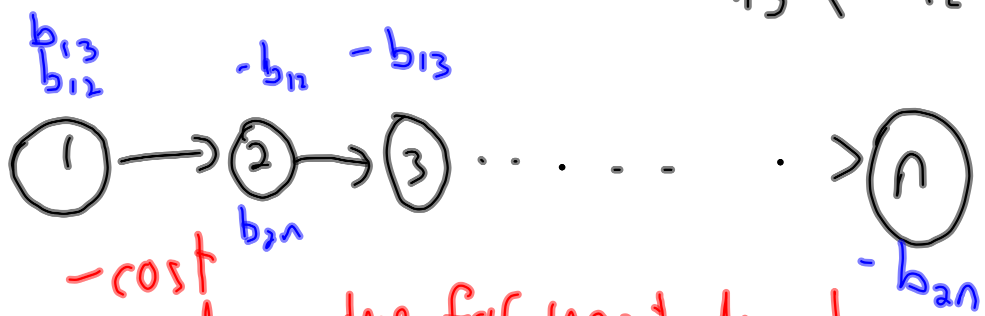
for each i, j $(i<j)$

b_{ij} - # of passengers who want to travel from i to j

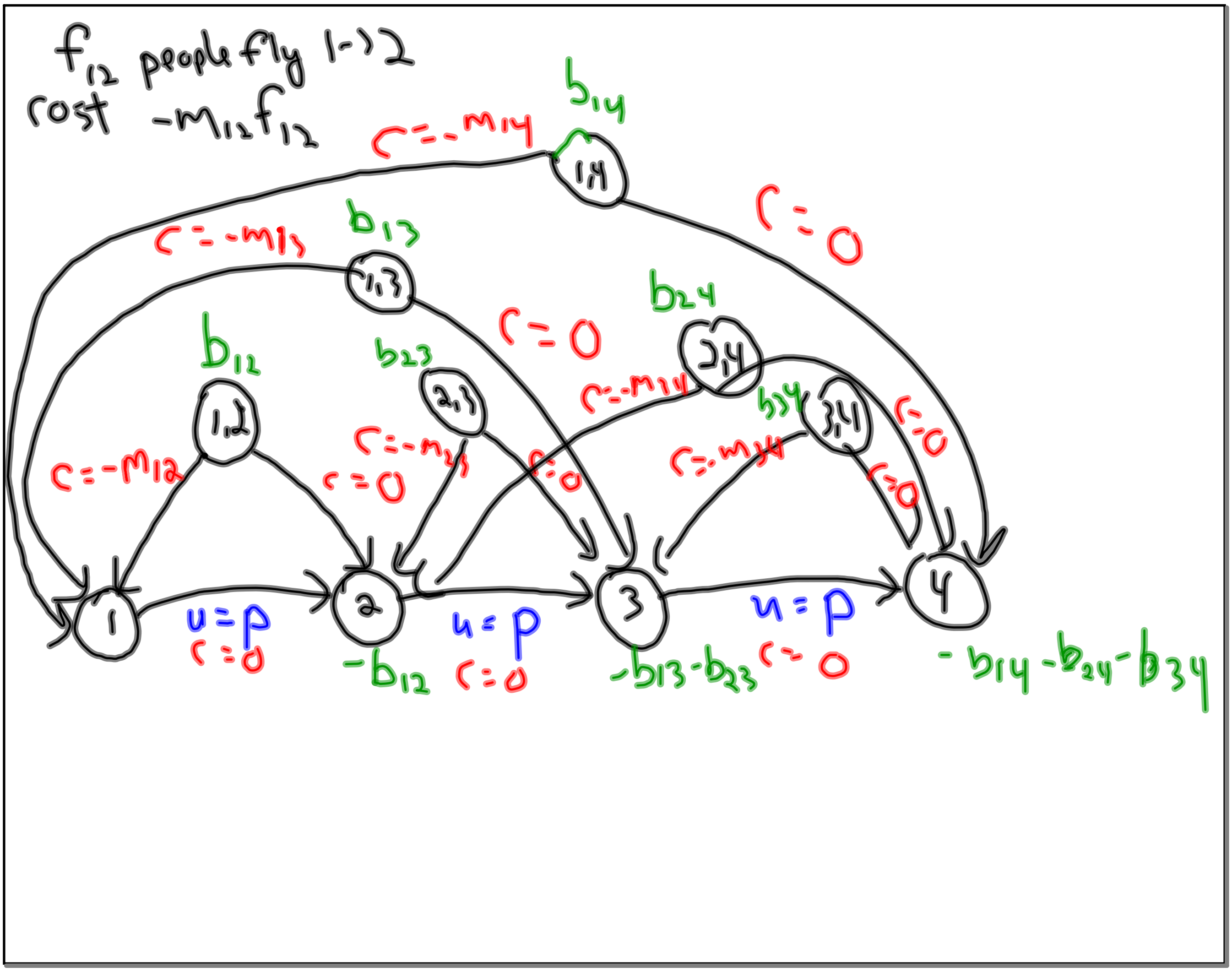
m_{ij} - payment for flying from i to j

$m_{13} \neq m_{12} + m_{23}$

$p = 5$
$b_{13} = 3$
$b_{24} = 3$



- cost
- need somewhere for unsat. demand to go
- max payments

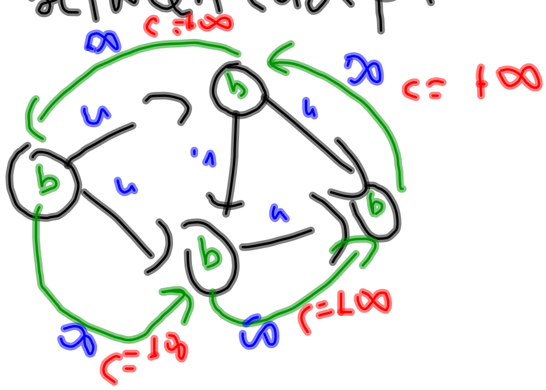


f_{12} people fly 1->2
cost $-m_{12} f_{12}$
b_{14}
$c = -m_{14}$
$c = 0$
b_{13}
$c = -m_{13}$
$c = 0$
b_{24}
b_{12}
b_{23}
$c = -m_{24}$
b_{34}
$c = -m_{12}$
$c = -m_{23}$
$c = 0$
$c = 0$
$c = -m_{34}$
$c = 0$
$c = 0$
$u = p$
$c = 0$
$u = p$
$c = 0$
$u = p$
$c = 0$
$-b_{12}$
$-b_{13} - b_{23}$
$-b_{14} - b_{24} - b_{34}$

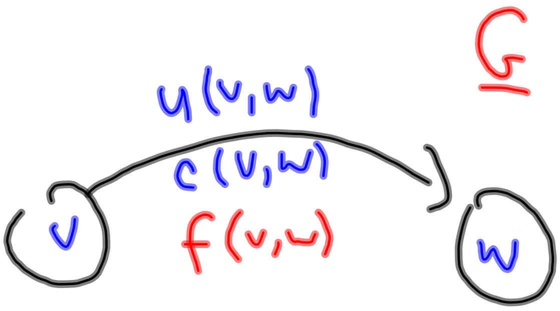
Assumptions

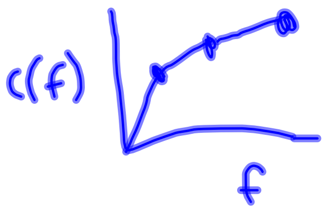
- $\sum b(v) = 0$
- Graph is directed
- costs/cap integral
- $\exists$ directed path of infinite capacity between each pair of nodes

if $(v,w) \in G$, $(w,v) \notin G$.

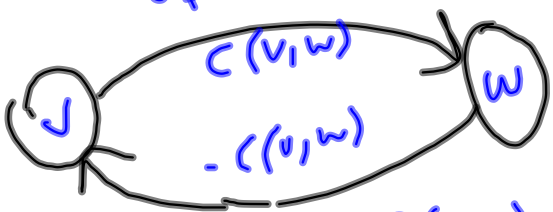


Residual Graph





$$u_f(v,w) = u(v,w) - f(v,w)$$



$$u_f(w,v) = f(v,w)$$

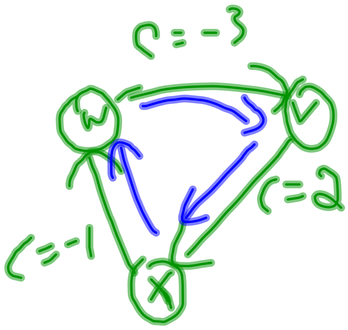
Optimality of a flow:

<u>Lemma</u>

A feasible flow f is optimal iff G_f has no negative cost cycles.

<u>Pf</u>

$\Rightarrow$ c.p. if G_f has a neg. cost cycle then f is not optimal

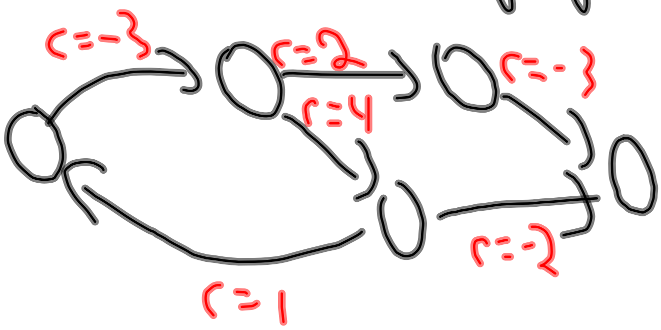


send 1 unit around cycle

flow is still feasible

$c \cdot f$ decreases by 2

$\Leftarrow$ if a flow has no neg cycles in G_f



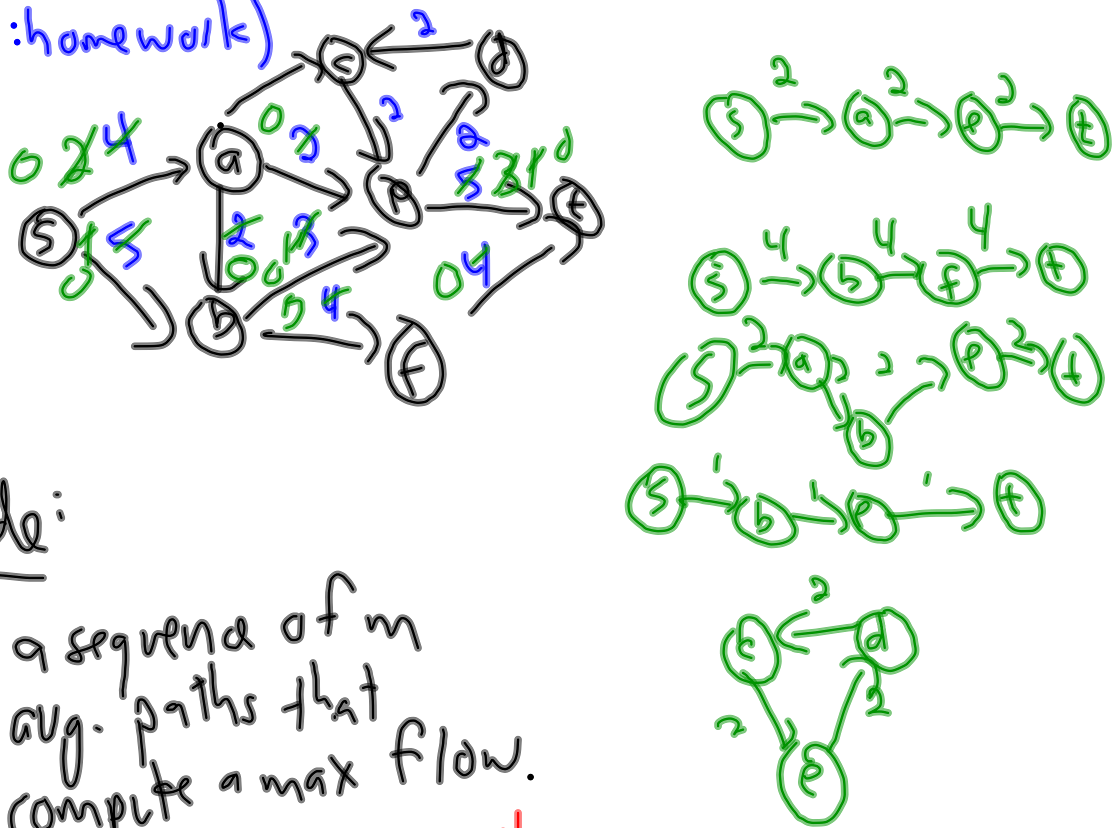
No way to push flow along a cycle
and decrease the cost

flow decomposition

For max flow:

Given any flows f, f', f' can be obtained from f via $\leq m+n$ augmenting paths & cycles

(Pf: homework)



Aside:

$\exists$ a sequence of m avg. paths that compute a max flow.

Algs uses $O(nm)$ avg. paths

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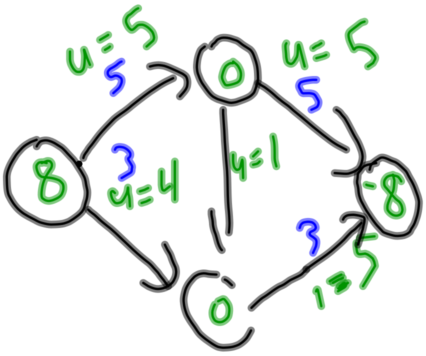
For min cost flow:

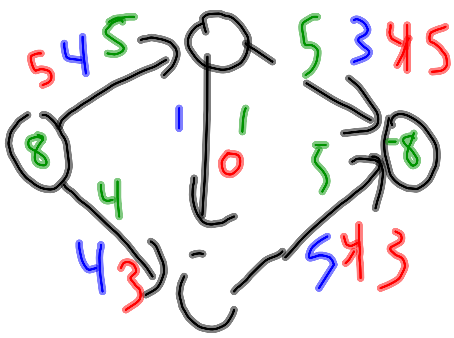
all feasible flows satisfy the constraints.

$\Rightarrow$ difference between any two flows is a collection of cycles

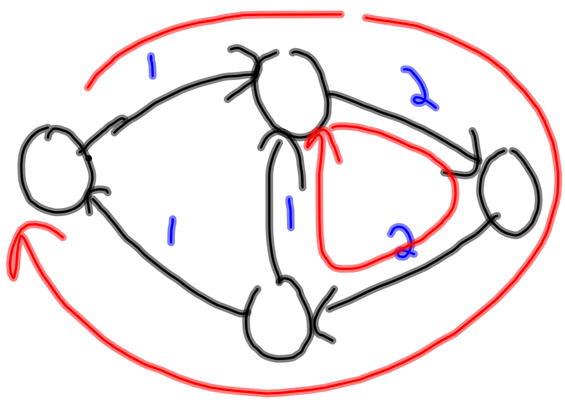
f



f'



$f - f'$



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