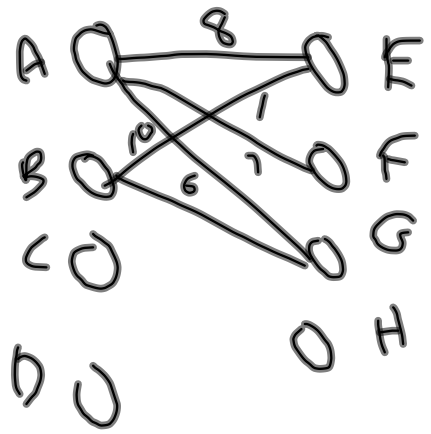
Stable Marriage



perfect matching

$\max \sum$ matched edges

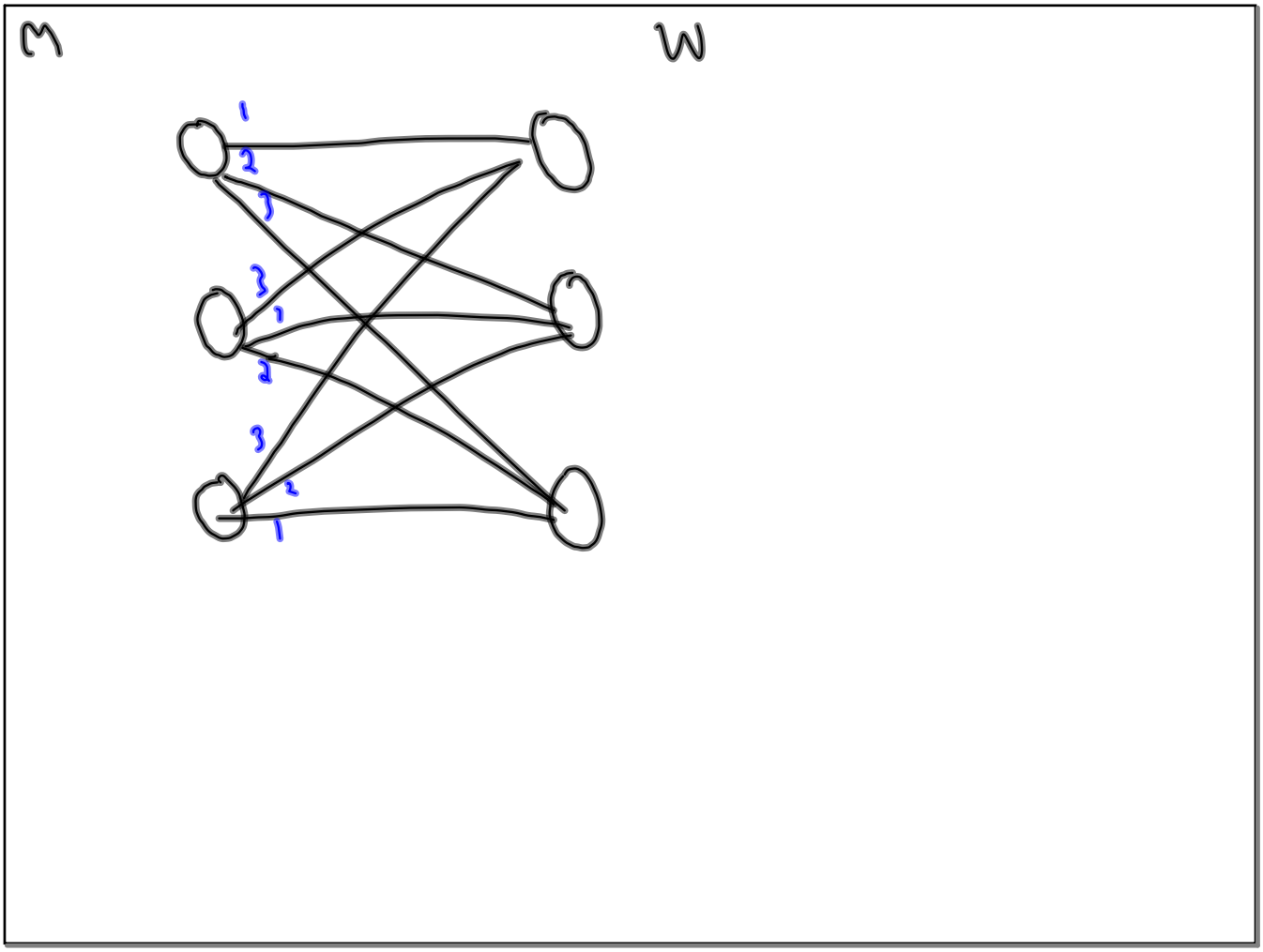
max (min (matched edges))

assignment problem

For any sets of preference lists,
a stable marriage exists
and can be found in $O(n^2)$ time

Alg.

- In each round, any unmatched man proposes to his highest ranked woman who has not yet rejected him.
- Women accept a new proposal if it is better than their current matching



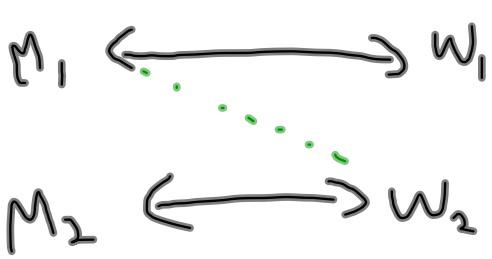
Apr 16-5:11 PM

Properties of Alg. (not life)

- once a woman is matched, she stays matched
- the partner of a woman only improves over time
- once a woman rejects a man, she would always reject him again in the future.
- a woman always accepts her first proposal

⇒ get a matching

Matching is stable
unstable



M_1 & W_2 prefer each other to their current partners

$\Rightarrow$ M_1 proposes to W_2
& W_2 rejected M_1,
$\Rightarrow$ W_2 was with M' who she preferred to M_1
but then she wouldn't have accepted M_2,
contradiction.

Among all stable marriages, this alg ~~finds~~
is man-optimal
& woman-pessimal.

- Each man is with the highest ranked woman he could be paired w/ in any stable marriage.

- Each woman is w/ the lowest ranked man