Stable Marriage

perfect matching

\[
\max \sum \text{matched edges}
\]

\[
\max (\min (\text{matched edges}))
\]

assignment problem
For any sets of preferences, a stable marriage exists and can be found in $O(n^2)$ time.

**Alg.**
- In each round, any unmatched man proposes to his highest ranked woman who has not yet rejected him.
- Women accept a new proposal if it is better than their current matching.
Properties of Alg. (not life)

- once a woman is matched, she stays matched
- the partner of a woman only improves over time
- once a woman rejects a man, she would always reject him again in the future.
- a woman always accepts her first proposal

⇒ get a matching
Matching is stable

\[ \text{unstable} \]

\[ M_1 \leftrightarrow W_1 \]
\[ M_2 \leftrightarrow W_2 \]

\[ M_1 \text{ proposes to } W_2 \]
\[ \text{and } W_2 \text{ rejected } M_1 \]
\[ \Rightarrow W_2 \text{ was with } M' \text{ who she preferred to } M_1 \]
\[ \text{but then she couldn't have accepted } M_2, \text{ contradiction.} \]
Among all stable marriages, this algorithm finds
is man-optimal
and woman-optimal.

- Each man is with the highest ranked woman
  he could be paired with in any stable marriage.

- Each woman is with the lowest ranked man.