\[ \leq \lg n \text{ iterations} \]

1 iteration takes \( O(m) \) time

\( O(m \lg n) \)

1 iteration

\# nodes halved

\# edges \?
If the edges and vertices halve

\( O(m) \) algorithm

\[
T(n,m) = T\left(\frac{n}{2}, \frac{m}{2}\right) + O(m + n)
\]

\[
= O(m + n).
\]

Identify non-MST edges quickly
Forest $F$

Let $w_F(u,v) = \max$ wt. edge on the path from $u$ to $v$ in $F$. ($\infty$ if none)

An $F$-heavy edge is not in the MST

Find any $F$, we can use to eliminate edges not in MST

$O(m)$ time you can find all $F$-heavy edges for a given $F$. 
MST(G) \ T(n, m)

1. Run 3 phases of Borouwka to get G1 with free edges E1, \( O(m) \)  
   MST has \( \frac{n}{8} \) nodes, \( \frac{m}{2} \) edges

2. Form H by including each edge of G1 with \( p = \frac{1}{2} \)  
   Compute F = MST(H) \( T\left(\frac{n}{8}, \frac{m}{2}\right) \)

3. Find the F-heavy edges of G1, delete them to get G2  
   Claim G2 has \( \frac{n}{8} \) nodes, \( \frac{m}{4} \) edges (in expectation)

4. \( F' = MST(G2) \) \( T\left(\frac{n}{8}, \frac{m}{4}\right) \)

5. Return \( F' \cup E \)  
   \( T(n, m) \leq T\left(\frac{n}{8}, \frac{m}{2}\right) + T\left(\frac{n}{8}, \frac{m}{4}\right) + O(m) = O(m) \)
Claim: Let $H$ be a subgraph of $G$ with edge $e$ chosen with probability $\frac{1}{2}$.
Let $F$ be a $\text{MSF}(H)$.

$$E\left[\sum \text{# of } F\text{-light edges in } G\right] \leq 2\eta$$

1000 nodes
500,000 edges
498,000 edges are $F$-heavy