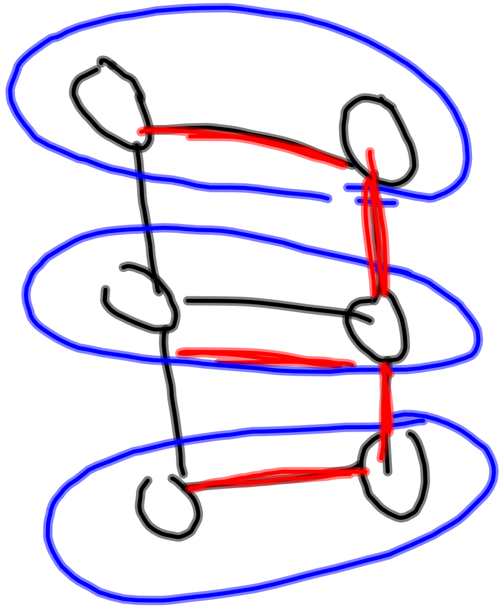


$\le \lg n$ iterations

1 iteration takes $O(m)$ time

$O(m \lg n)$

1 iteration

\# nodes halved

\# edges ?

If # edges & vertices halve

$O(m)$ algorithm

$$T(n,m) = T\left(\frac{n}{2}, \cancel{\frac{m}{2}}^{\,m-\frac{n}{2}}\right) + O(m+n)$$

$$= O(m+n).$$

identify non-MST edges quickly

Forest F

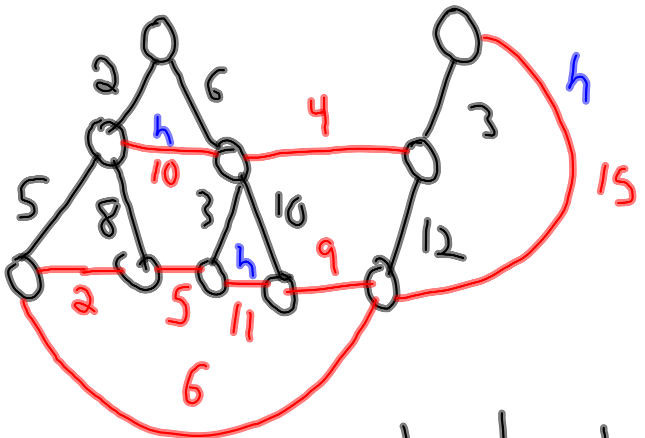
let $w_F(u,v)$ = max wt. edge on the path from u to v in F. (∞ if none)

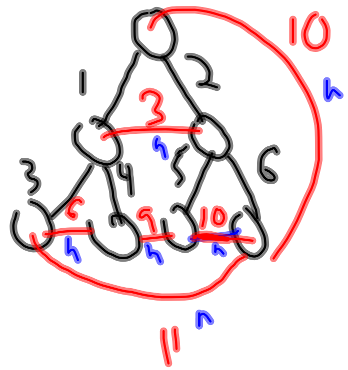
edge (u,v) is F-heavy if $w(u,v) > w_F(u,v)$

F-light o.w.

An F-heavy edge is not in the MST

F





Find any F, we can use to eliminate edges not in MST

$O(m)$ time you can find all F-heavy edges for a given F.

MST(G) $T(n,m)$

1. Run 3 phases of Borouvka to get G_1 w/ tree edges E_1 $O(m)$

$\le \frac{n}{4}$ nodes

$\le m$ edges

MST

H has $\frac{n}{8}$ nodes: $\frac{m}{2}$ edges

2. Form H by including each edge of G_1 w/ $P_r = \frac{1}{2}$
   Compute $F = MST(H)$ $T(\frac{n}{8}, \frac{m}{2})$
3. Find the F-heavy edges of G_1 delete them to get G_2

Claim G_2 has $\frac{n}{8}$ nodes $\frac{n}{4}$ edges (in expectation)

4. $F' = MST(G_2)$ $T(\frac{n}{8}, \frac{n}{4})$
5. Return $F' \cup F$

$$T(n,m) \le T\left(\frac{n}{4}, \frac{m}{2}\right) + T\left(\frac{n}{8}, \frac{\cancel{n}^{\,m}}{4}\right) + O(m) = O(m)$$

Claim Let H be a subgraph of G w/ edge
edge chosen w/ $P_r = \frac{1}{2}$
Let F be a MSF(H)
$E[\#$ of F-light edges in G$] \leq 2n$

1000 nodes
500,000 edges
498,000 edges
are F-heavy