

Proof

Think about Kruskal's alg.

G

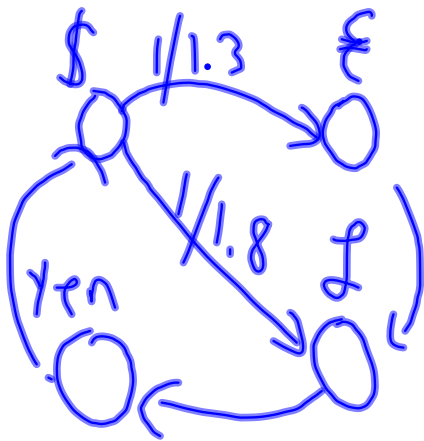
e_1	e_2	e_3	e_4	e_5	e_6	e_7	...	e_m
H	X	X	H	H	X	X	H	H
in F			in F	not in F			in F	not in F
F-l	<u>F-l</u>	<u>F-l</u>	F-l	F-h	E-h	E-h	F-l	F-h + F-l...



An edge is F-heavy ind. of P
random choice

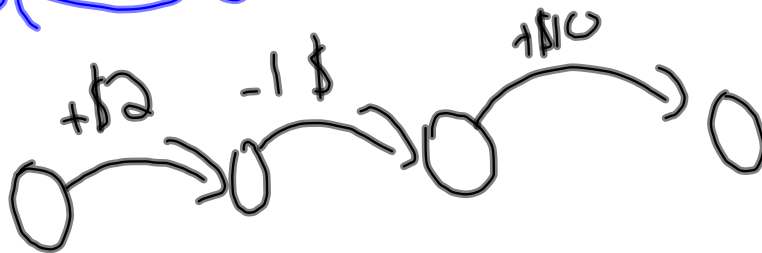
edges not in H, but F-light

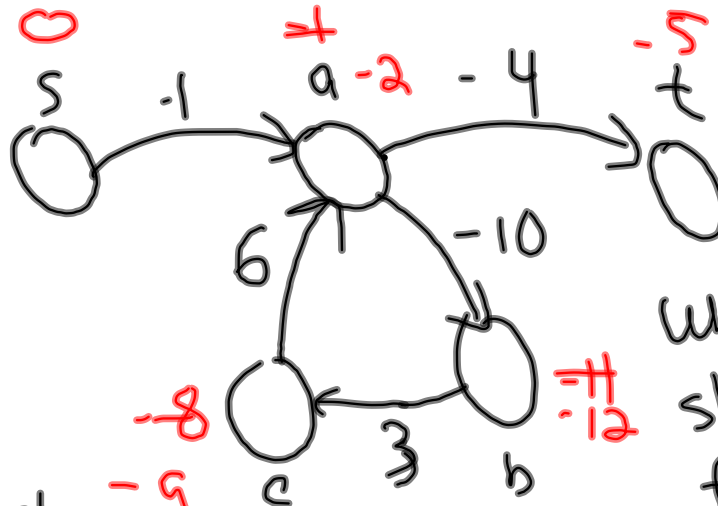
Spanning tree F is formed by, for each F-light edge, flip a coin, if heads, add to tree else don't
continue until we've added $n-1$ edges to F.
 \Rightarrow we only process $2(n-1)$ F-light edges (in exp.)



$$-\log(r_1 r_2 r_3) \stackrel{!}{>} 0$$

$$-\log r_1 + -\log r_2 + -\log r_3 < 0$$





$\begin{matrix} \rightarrow x \\ y \\ \leftarrow y \\ z \end{matrix}$
 neg. cycle
 & nothing to relax

What is the shortest path from s to t?

Sabracabcabcabc...
 NOT WELL DEFINED

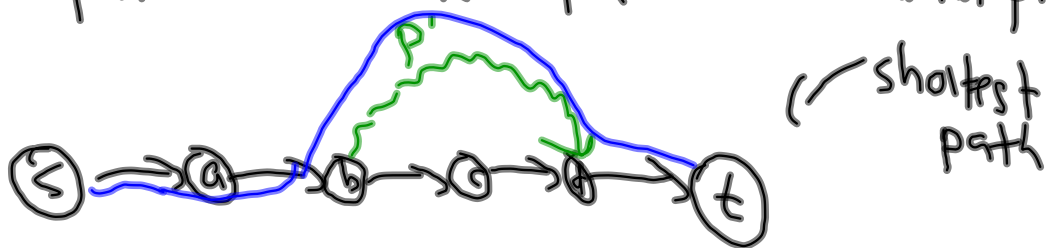
$$\begin{cases} d(y) \leq d(x) + w(x,y) \\ d(z) \leq d(y) + w(y,z) \\ d(x) \leq d(z) + w(z,x) \end{cases}$$

$$\Leftrightarrow \begin{cases} \cancel{d(y)} - \cancel{d(x)} \leq w(x,y) \\ \cancel{d(z)} - \cancel{d(y)} \leq w(y,z) \\ \cancel{d(x)} - \cancel{d(z)} \leq w(z,x) \end{cases}$$

$$0 \leq w(x,y) + w(y,z) + w(z,x)$$

cycle is not negative. contr.

Subpaths of shortest paths are shortest paths



Proof Suppose not, suppose for example, that $b-c-d$ is not the shortest path from b to d .
 & P' is shorter. P'
 Then $s-a-b \overset{P'}{\curvearrowright} d-t$ is shorter
 than $s-a-b-c-d-t$, a contradiction.

