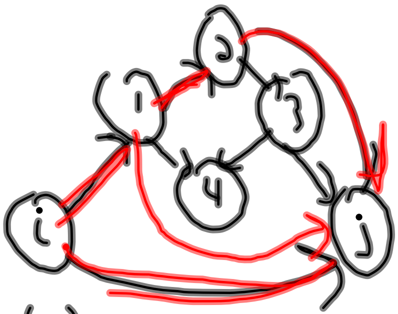


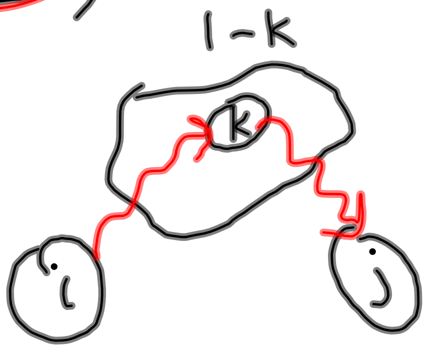
$d_{ij}^{(2)}$

goes thru k

$$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$$

doesn't go thru k

$$d_{ij}^{(k-1)}$$



Johnson's Algorithm

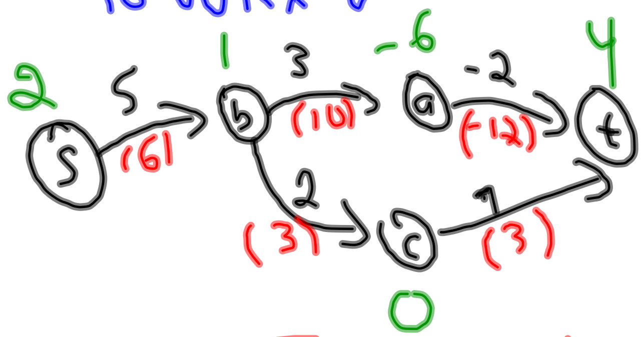
Graph w/ arbitrary edge weights

Repeated single source $(V^2 E)$ V^3

If non-neg $O(VE + V^2 \lg V)$

- Run BF once
- Convert to an equivalent problem w/ non-neg edge wts.
→ Run Dijkstra V-1 time

Reweighting: suppose I assign a price $p(v)$ to vertex v



c_{ij} - cost ij

p_i price at i

reduced cost $\bar{c}_{ij} = c_{ij} + p_i - p_j$

s-t path q: $\sum_{(i,j)\in q} \bar{c}_{ij} = \bar{c}_{12} + \bar{c}_{23} + \bar{c}_{34}$

$= c_{12} + p_1 - \cancel{p_2} + c_{23} + \cancel{p_2} - \cancel{p_3} + c_{34} + \cancel{p_3} - p_4$

the cost of any s-t path increases by $p_s - p_t$ w.r.t. red. cost

Computing shortest paths w.r.t. reduced costs return the same paths as w.r.t. orig. costs, for any prices p

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Choose prices p s.t. $\bar{c}_{ij} \geq 0 \quad \forall (i,j) \in E$.

Recall s.p. optimality conditions

for (i,j) $\quad d_j \leq d_i + c_{ij}$ $\quad$ $\textcircled{i} \xrightarrow{3} \textcircled{j}$ (4)

$$0 \leq c_{ij} + d_i - d_j$$

Choose $p = d$

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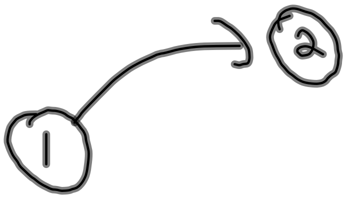
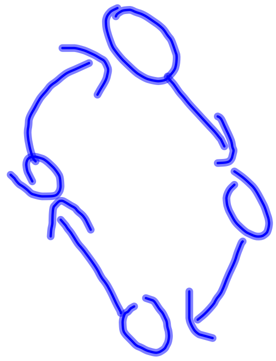




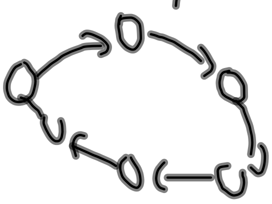
- For a cycle X, for any p

$$\sum_{(i,j)\in X} c_{ij} = \sum_{(i,j)\in X} \bar{c}_{ij}$$

- Suppose X is a neg. cost cycle, then $\nexists\, p$ s.t. $\bar{c}_{ij} \geq 0 \quad \forall (i,j)\in X$.

Minimum Mean Cycle

$$\mu(X) = \frac{\sum_{(i,j)\in X} c_{ij}}{|X|}$$



$|X|$ = # edges in X.

Find X that minimizes $\mu(X)$.

tramp steamer problem

Can't compute minimum value cycle (NP-hard)