Successive Shortest Paths for Minimum Cost Flow

Successive Shortest Path

$f = 0; \quad \Pi = 0$
$e(v) = b(v)\ \forall v \in V$
Initialize $E = \{v : e(v) > 0\}$ **and** $D = \{v : e(v) < 0\}$
while $E \neq 0$
    do Pick a node $k \in E$ **and** $\ell \in D$
        Compute $d(v)$**, shortest path distances from** k **in** G_f
          w.r.t. edge distances c^π**.**
        Let P **be a shortest path from** k **to** ℓ**.**
        Set $\pi = \pi - d$
        Let $\delta = \min\{e(k), -e(\ell), \min\{u_f(v,w) : (v,w) \in P\}\}$
        Send δ **units of flow on the path** P
        Update f**,** G_f**,** E**,** D **and** c^π**.**

Capacity Scaling Algorithm for Minimum Cost Flow

Successive Shortest Path

$f = 0; \quad \pi = 0$
$e(v) = b(v)\ \forall v \in V$
$\Delta = 2^{\lfloor U \rfloor}$
while $\Delta \geq 1$
  do **(**Δ **scaling phase)**
    for every edge $(v, w) \in G_f$
      do if $u_f(v, w) \geq \Delta$ **and** $c^\pi(v, w) < 0$
        then Send $u_f(v, w)$ **units of flow on** (v, w)**; update** f**,** e
    $S(\Delta) = \{v \in V : e(v) \geq \Delta\}$
    $T(\Delta) = \{v \in V : e(v) \leq -\Delta\}$
    while $S(\Delta) \neq 0$ **and** $T(\Delta) \neq 0$
      do Pick a node $k \in S(\Delta)$ **and** $\ell \in T(\Delta)$
        Compute $d(v)$**, shortest path distances from** k **in** $G_f(\Delta)$
          w.r.t. edge distances c^π**.**
        Let P **be a shortest path from** k **to** ℓ**.**
        Set $\pi = \pi - d$
        Let $\delta = \min\{e(k), -e(\ell), \min\{u_f(v, w) : (v, w) \in P\}\}$
        Send δ **units of flow on the path** P
        Update f**,** $G_f(\Delta)$**,** $S(\Delta)$**,** $T(\Delta)$ **and** c^π**.**
    $\Delta = \Delta/2$