Mid-term

March 22

(10,11) 11- 1 (1,2)

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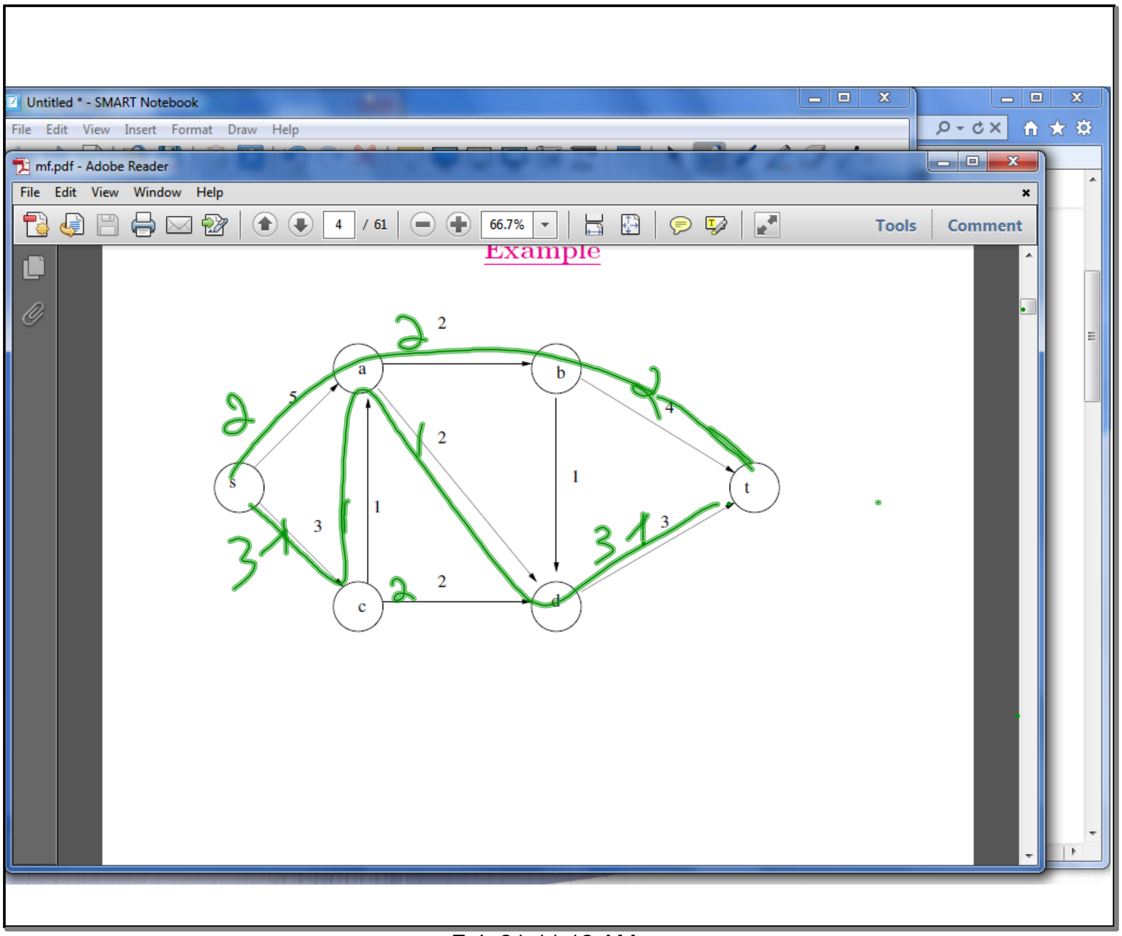

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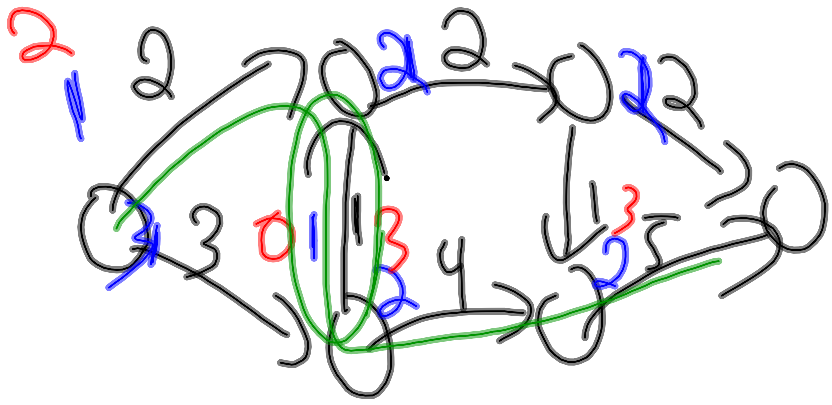

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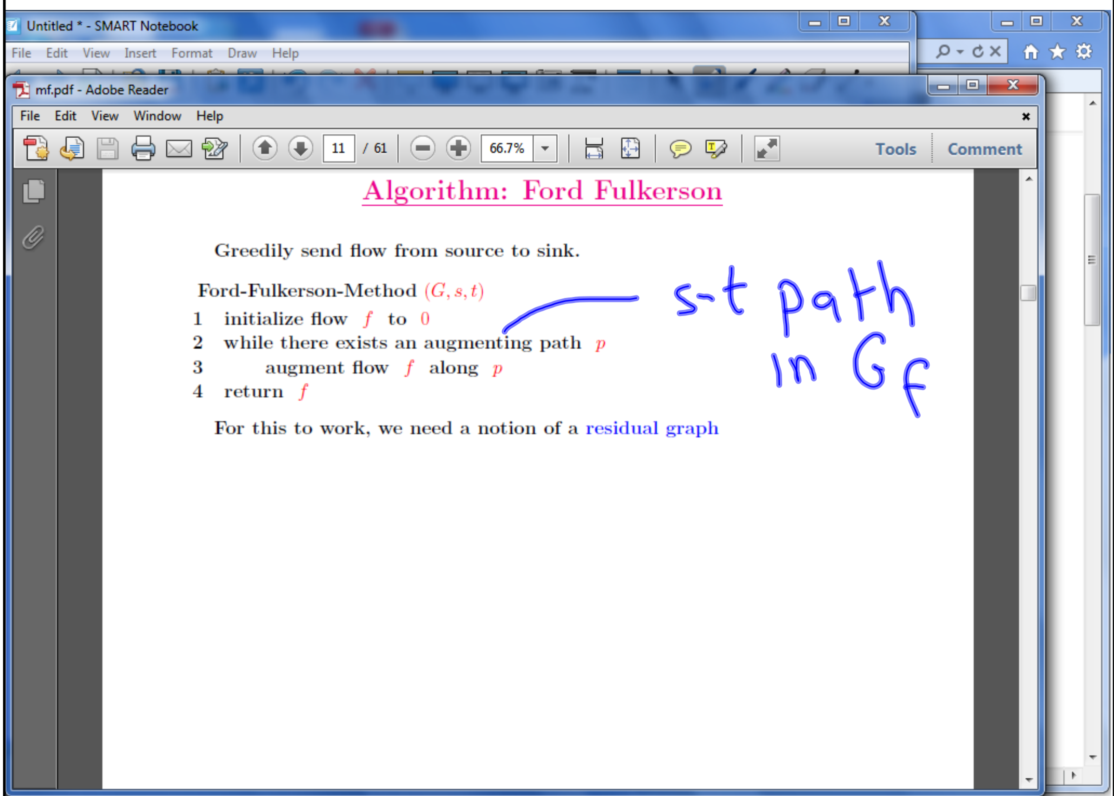

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Algorithm: Ford Fulkerson

Greedily send flow from source to sink.

Ford-Fulkerson-Method (G, s, t)

1 initialize flow f to 0
2 while there exists an augmenting path p
3 augment flow f along p
4 return f

s-t path in G_f

For this to work, we need a notion of a residual graph

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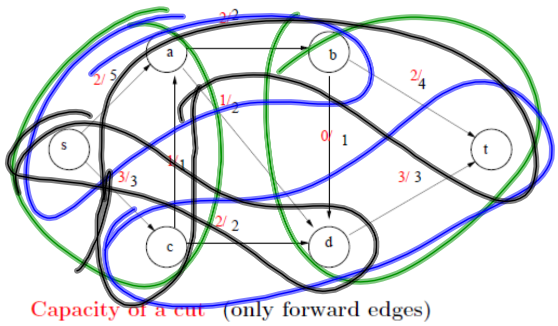
An s - t cut satsfies

- $s \in S$, $t \in T$
- $S \cup T = V$, $S \cap T = \emptyset$



Capacity of a cut (only forward edges)

$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$

Flow crossing a cut (net flow)

$$(S,T) = \sum_{u \in S, v \in T} f(u,v) - \sum_{u \in T, v \in S} f(u,v)$$

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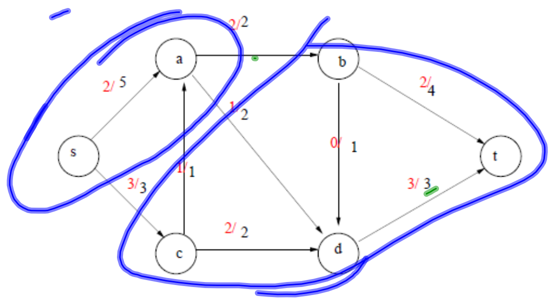
An s - t cut satsfies

- $s \in S$, $t \in T$
- $S \cup T = V$, $S \cap T = \emptyset$

S f = 5, c = 8



cap = 2 + 2 + 3 = 7

f = 2 + 1 · 1 + 3 = 5

(s, a, c, d)
cap = 5
flow = 5

Capacity of a cut (only forward edges)

$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$

Flow crossing a cut (net flow)

$$f \quad (S,T) = \sum_{u \in S, v \in T} f(u,v) - \sum_{u \in T, v \in S} f(u,v)$$

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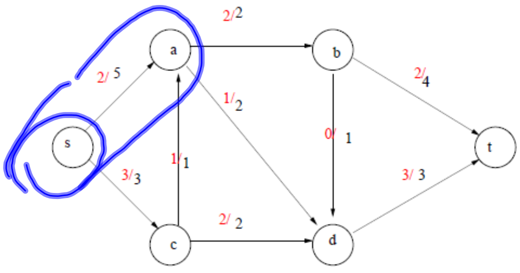
An s - t cut satsfies

- $s \in S$, $t \in T$
- $S \cup T = V$, $S \cap T = \emptyset$



S,c
−(S,a)
(a,b)
(a,d)
−(a,c)

Capacity of a cut (only forward edges)

$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$

Flow crossing a cut (net flow)

$$(S,T) = \sum_{u \in S, v \in T} f(u,v) - \sum_{u \in T, v \in S} f(u,v)$$

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Properties of cuts and flows

Capacity of a cut (only forward edges)

$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$

Flow crossing a cut (net flow)

$$f(S,T) = \sum_{u \in S, v \in T} f(u,v) - \sum_{u \in T, v \in S} f(u,v)$$

- For all cuts (S,T) and all feasible flows f , $f(S,T) \le c(S,T)$ (weak duality).
- For all pairs of cuts (S_1,T_1) and (S_2,T_2) , and all feasible flows f , $f(S_1,T_1) = f(S_2,T_2)$.

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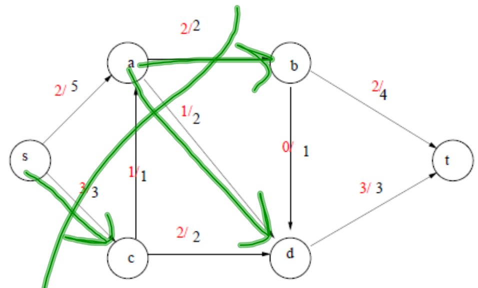
s - t Cuts

An s - t cut satsfies

- $s \in S$, $t \in T$
- $S \cup T = V$, $S \cap T = \emptyset$



Capacity of a cut (only forward edges)

$$c(S,T) = \sum_{u \in S, v \in T} c(u,v)$$

Flow crossing a cut (net flow)

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Max-flow min-cut theorem

If f is a flow in a flow network $G = (V,E)$ with source s and sink t , then the following conditions are equivalent:

1. f is a maximum flow in G .
2. The residual network G_f contains no augmenting paths.
3. $|f| = c(S,T)$ for some cut (S,T) of G .

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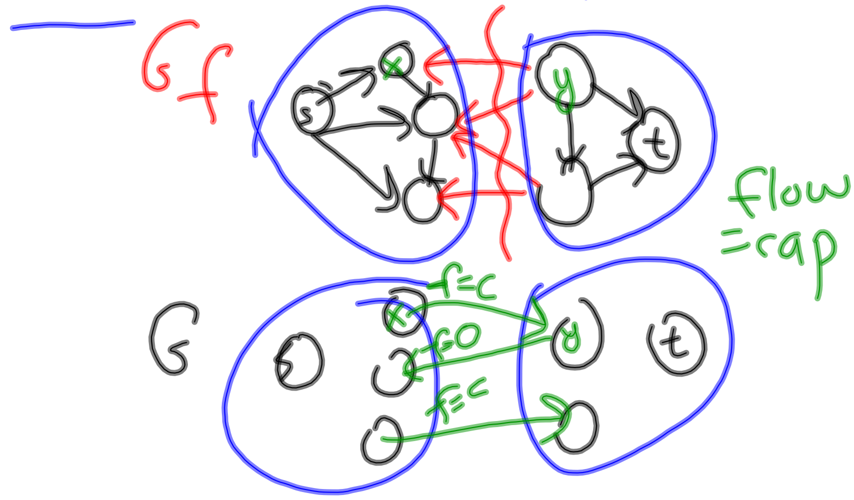
$\underline{1 \Rightarrow 2}$ $(\neg 2 \Rightarrow \neg 1)$ G_f has aug. paths $\Rightarrow$ f is not a max flow.

$\underline{2 \Rightarrow 3}$ G_f has no aug. paths



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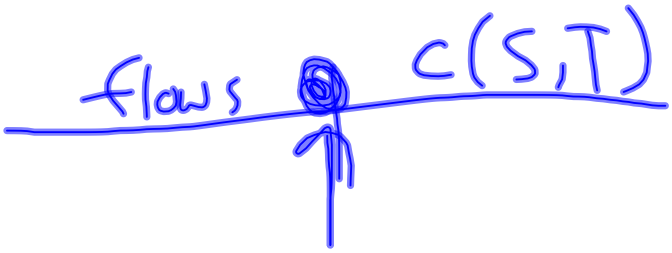
$\underline{3 \Rightarrow 1}$

$|f| = c(S,T)$



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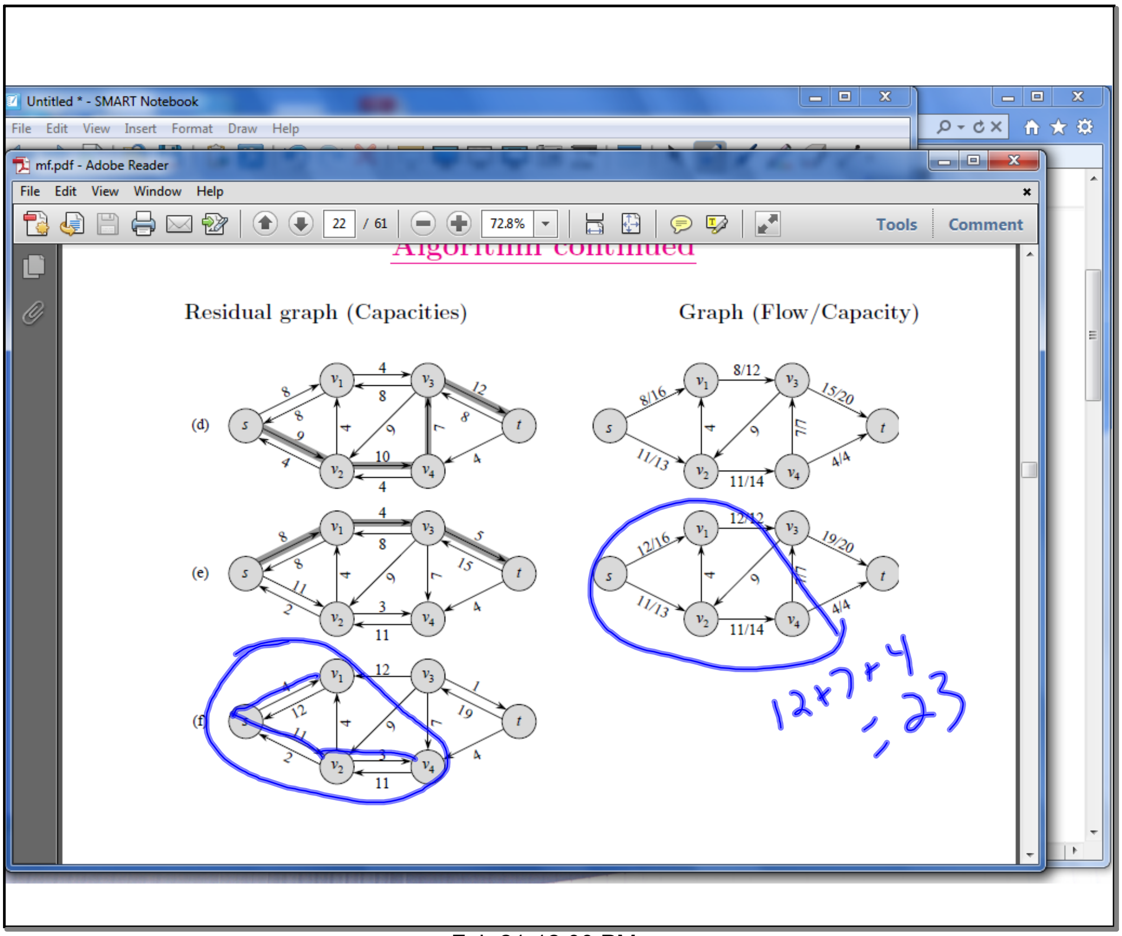

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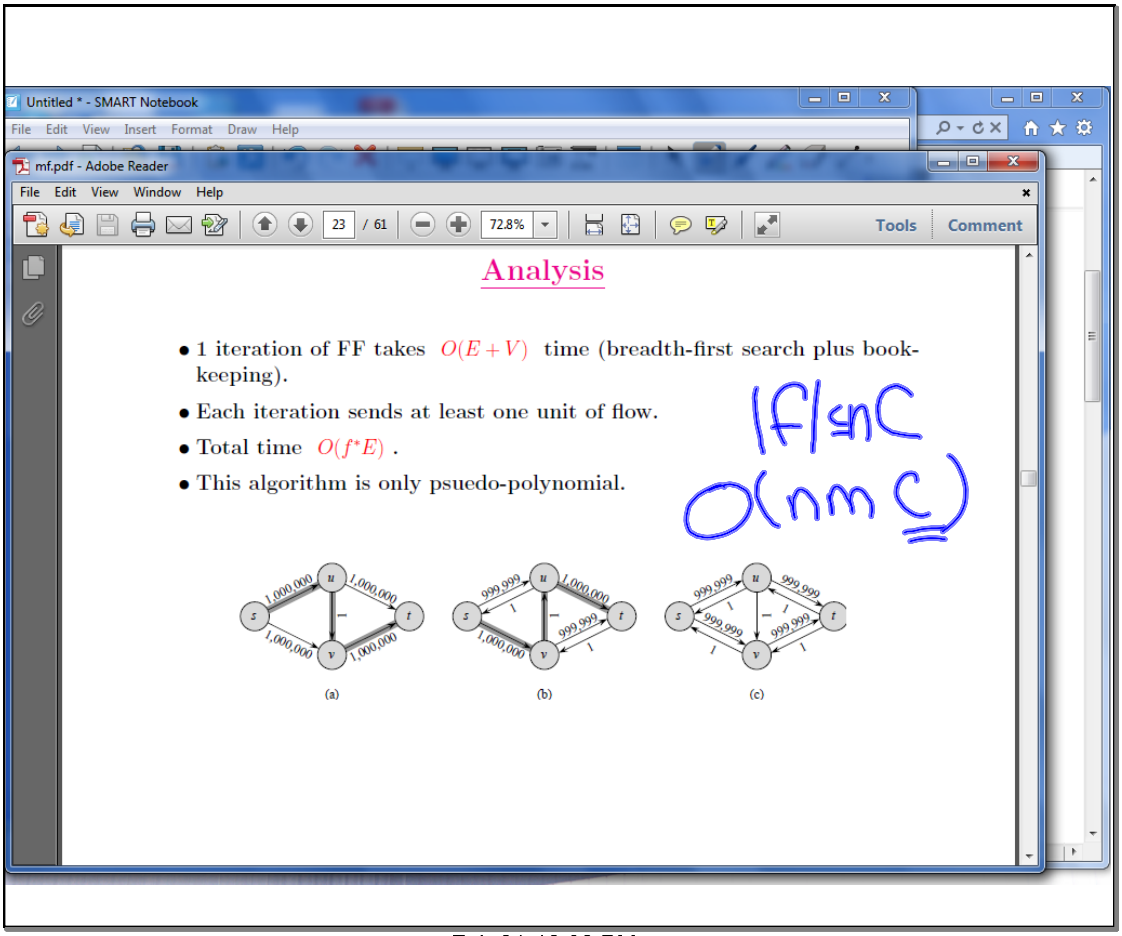

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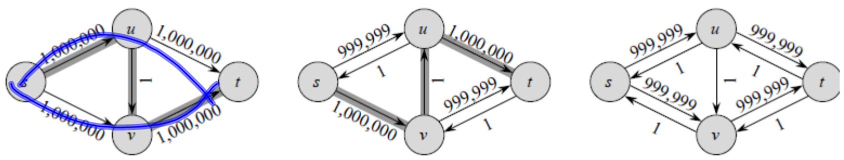
Each iteration sends at least one unit of flow.

Total time $O(f^*E)$.

This algorithm is only psuedo-polynomial.



(a) (b) (c)

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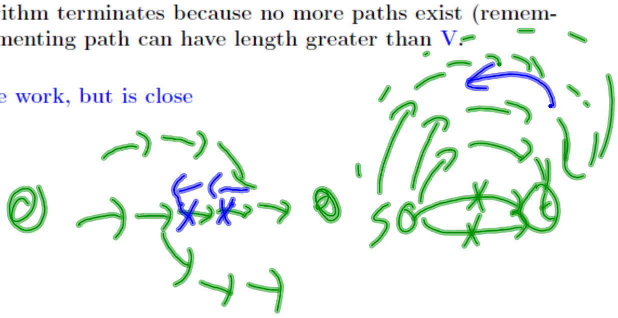
Idea for Analysis

Intuition

- Augment along shortest path in residual graph.
- Short paths get "saturated" and disappear.
- Future augmenting paths are along longer paths
- Eventually algorithm terminates because no more paths exist (remember that no augmenting path can have length greater than V.

This doesn't quite work, but is close



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