
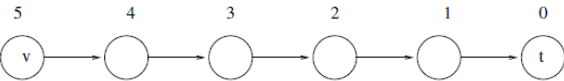


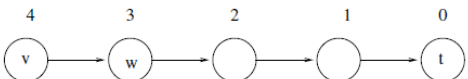
How could this happen?



Before



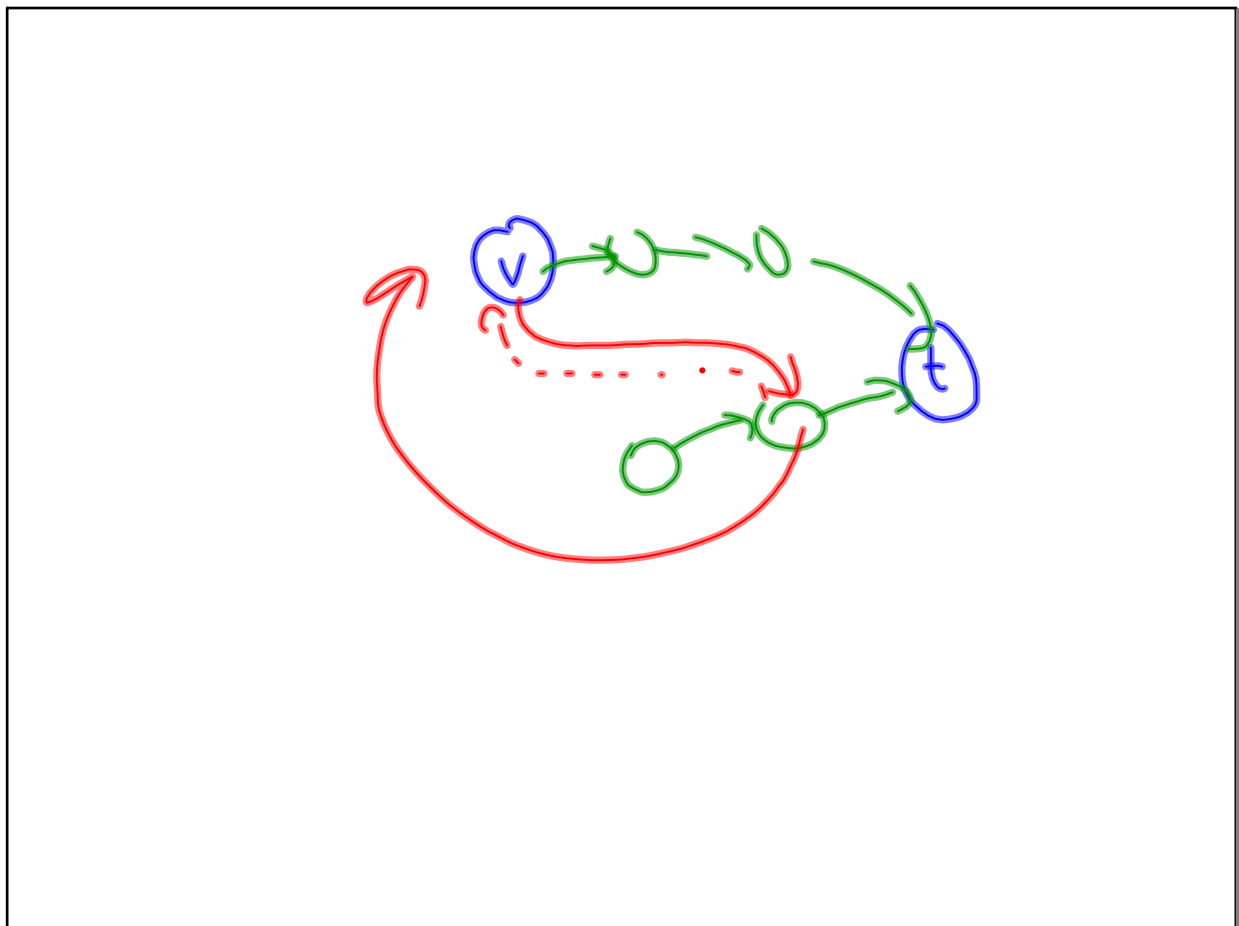
After



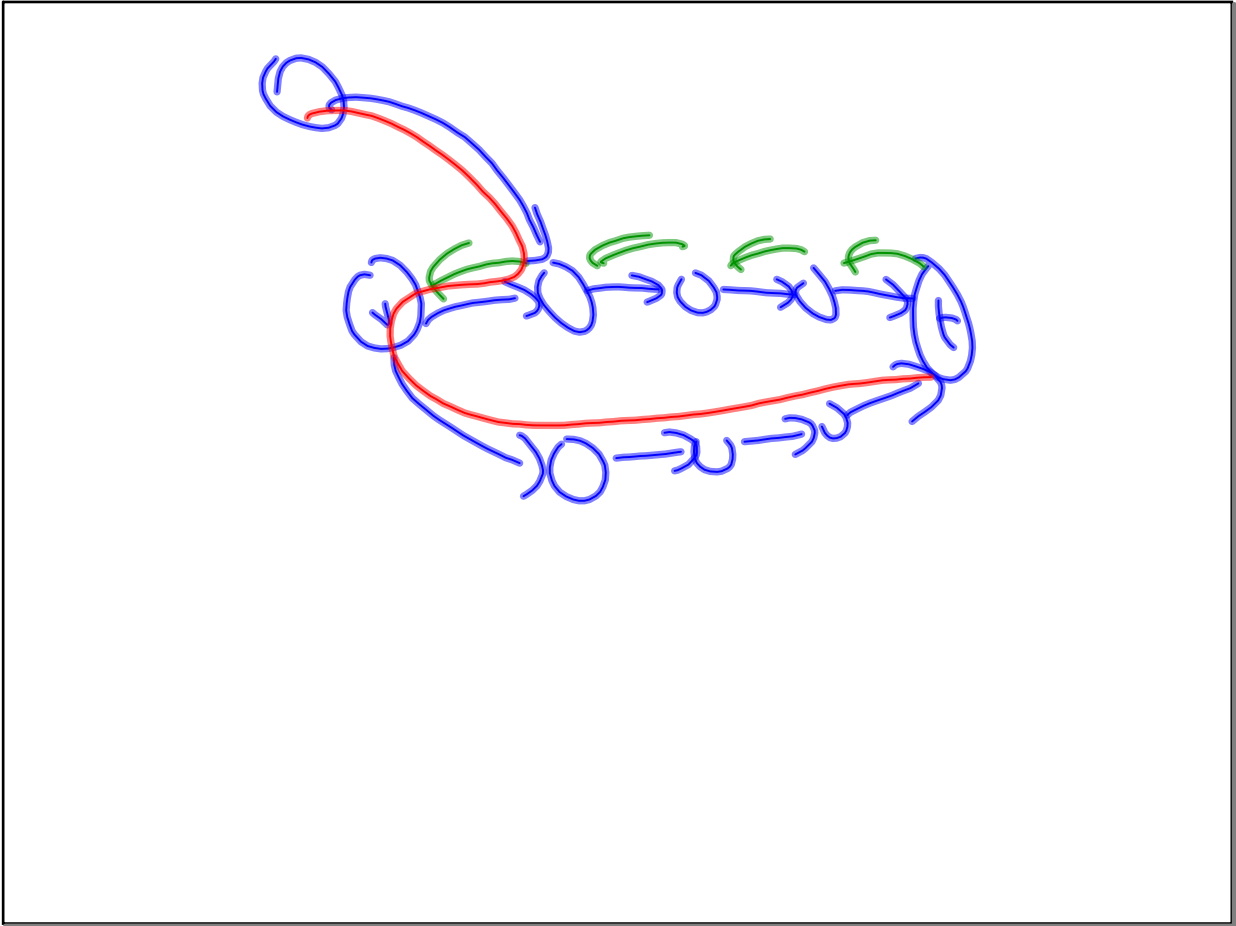
- $(v, w) \in G'_f$ but not in G_f . (why?)
- We must have sent flow on (w, v) in the augmenting path
- $\delta(w) = 6$
- $\delta'(w) < \delta(w)$, which contradicts v being the minimum vertex whose label decreased.

Conclusion: $\delta(v)$ never decreases.

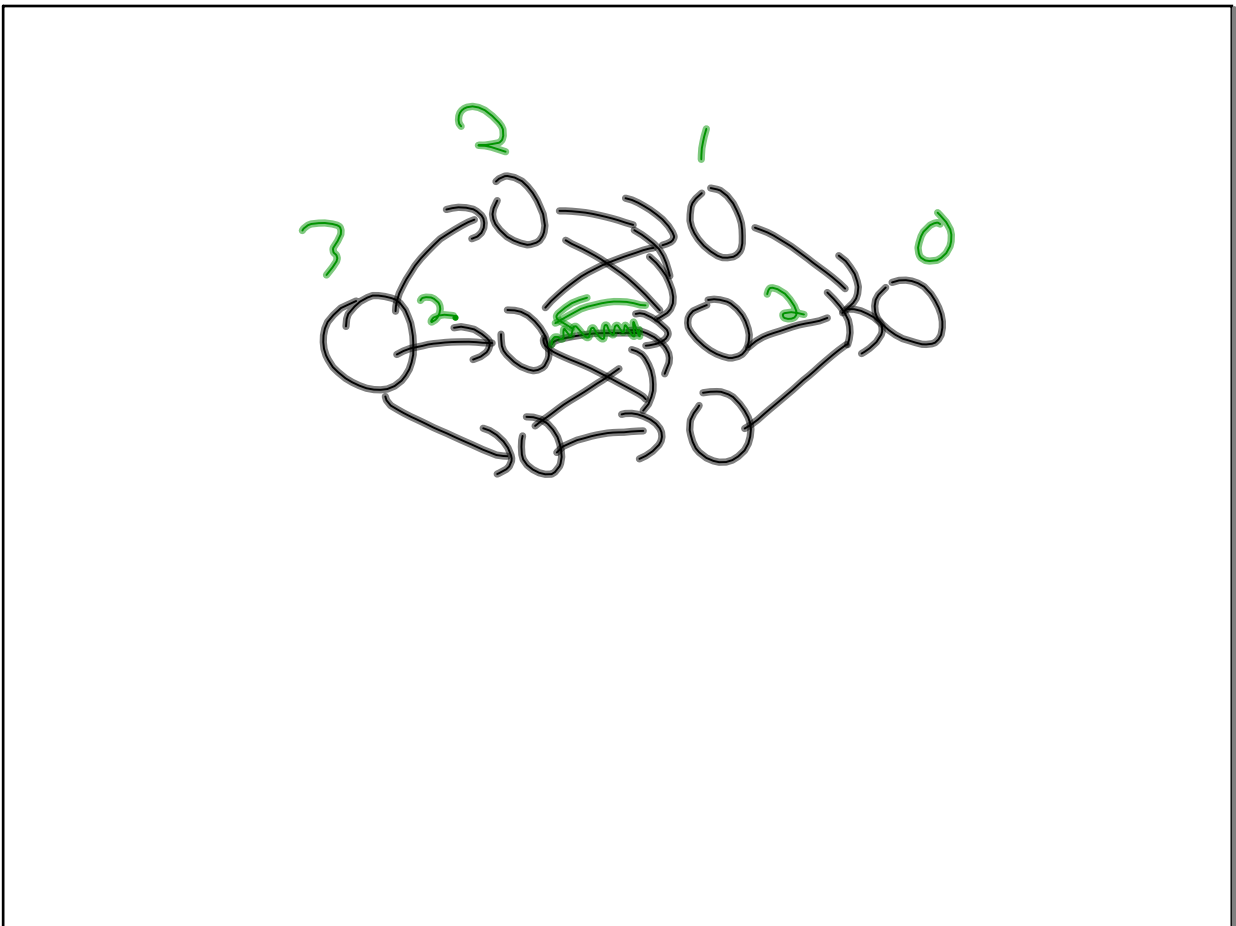
Feb 23-11:00 AM



Feb 23-11:21 AM



Feb 23-11:22 AM



Feb 23-11:25 AM

Events Between 2 edge saturations

saturate (v,w)

saturate (w,v)

saturate (v,w)

- Both v and w have to be relabeled.
- So each edge can be saturated at most $V/2$ times.

Feb 23-11:39 AM

Scaling of Capacities

The left diagram shows a network with nodes s, a, b, t . Edges and their flow/capacity values are: $s \rightarrow a$ (4/4), $s \rightarrow b$ (2/2), $a \rightarrow b$ (2/3), $a \rightarrow t$ (5/6), $b \rightarrow t$ (1/1). Red numbers above nodes are: $s: 100, a: 110, b: 010, t: 001$.

The right diagram shows a scaled network with nodes s, a, b, t . Edges and their flow/capacity values are: $s \rightarrow a$ (1/1), $s \rightarrow b$ (2/2), $a \rightarrow b$ (3/3), $a \rightarrow t$ (1/1), $b \rightarrow t$ (1/1). A blue edge is shown between s and t .

Feb 23-11:50 AM

Repeat

- double current flow
- add a bit of capacity
- solve max flow problem

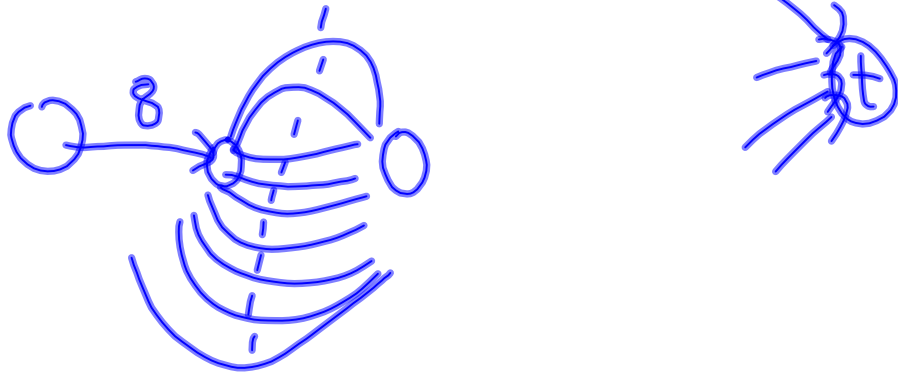
$\lg C$ iterations

Feb 23-11:56 AM

- doubling flow, add a bit of capacity maintains feasibility
- last iteration computes a max flow

Feb 23-11:59 AM

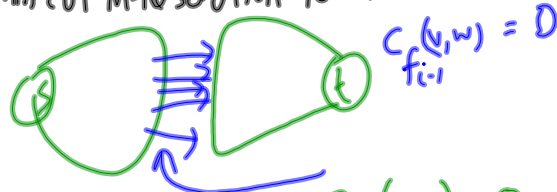
If you started an iteration w/
 $C_f(v,w) \leq 1 \quad \forall (v,w)$
 immediate bound of n iterations



Feb 23-12:03 PM

Claim
 let G_f be the res. graph at the beginning
 of a scaling iteration i . let f' be
 a max flow in G_f . Then $|f'| \leq M$.

Pr
 Look at min cut X to solution to iteration $i-1$.



At the beginning of iter i . $C_{f_i}(v,w) = 0$ or 1

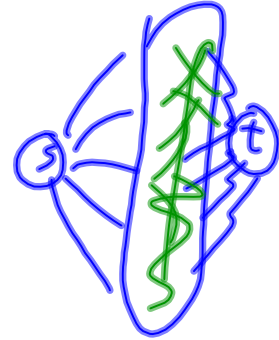
- \Rightarrow For cut X , the total res. cap. $\leq M$
- \Rightarrow min cut " " " " $\leq M$
- \Rightarrow \max flow in $G_{f_i} \leq M$ \otimes
- \Rightarrow need $\leq n$ avg. paths

Feb 23-12:06 PM

$$O(m^2 \lg u)$$

careful

$$O(mn \lg u)$$



Feb 23-12:12 PM