

Bounding R and S

$$\Phi = \sum_{v:e(v)>0} d(v)$$

O<sub>2,10</sub>

**Relabellings**

- Each relabelling must increase  $\Phi$  by at least 1.
- Total increase in  $\Phi$  associated with relabelling  $v$  is at most  $2V$ .
- Total increase associated with all relabellings is at most  $2V^2$ .

**Saturating Pushes**

- A saturating push leaves excess at  $v$ . It adds excess to  $w$ .
  - If  $w$  already had excess, then  $\Phi$  is unchanged.
  - If  $w$  did not have excess, then  $\Phi$  increases by  $d(w)$ , which is at most  $2V$ .
- There are at most  $O(EV)$  saturating pushes, therefore total increase due to saturating pushes is  $O(EV^2)$ .

Mar 1-10:57 AM

Bounding R and S

$$\Phi = \sum_{v:e(v)>0} d(v)$$

**Relabellings**

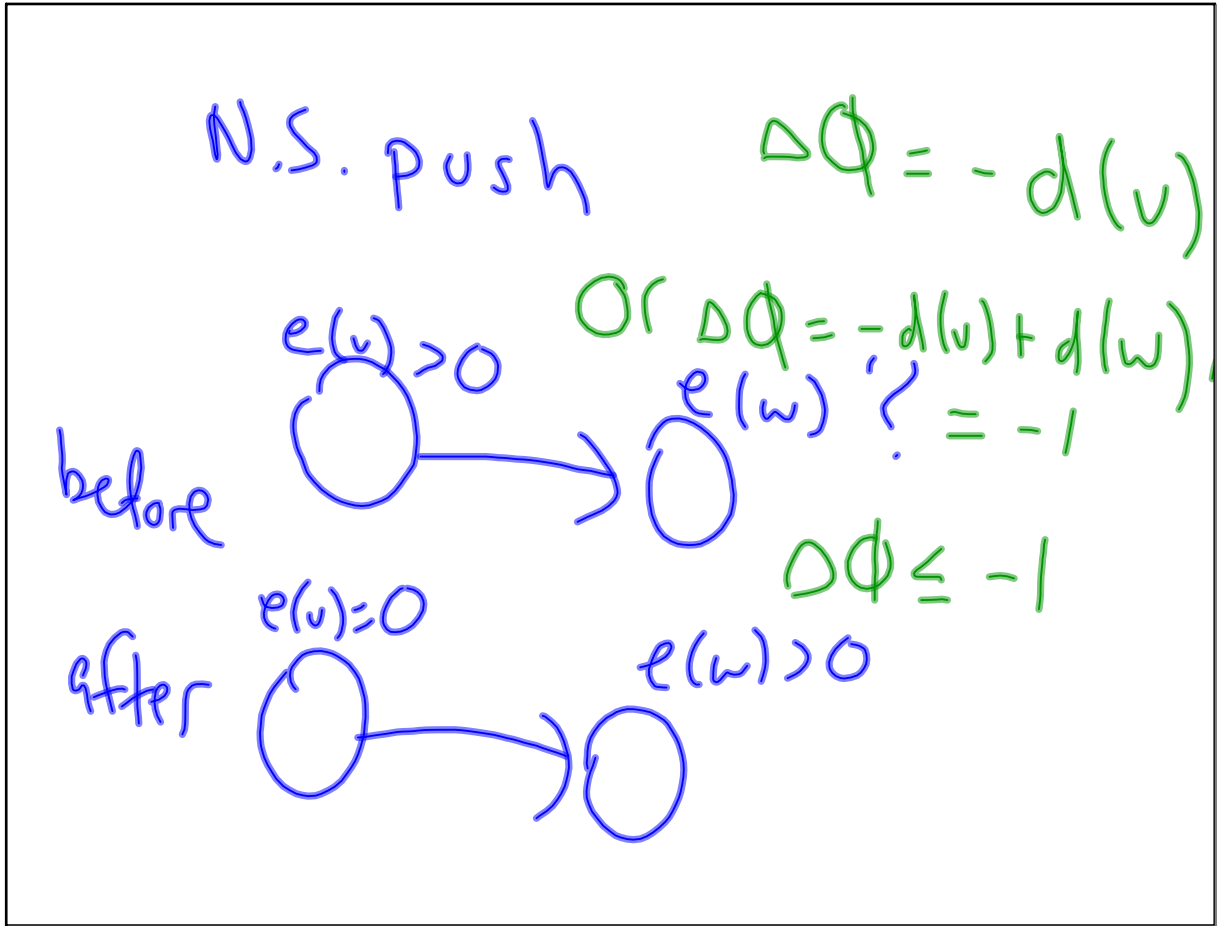
- Each relabelling must increase  $\Phi$  by at least 1.
- Total increase in  $\Phi$  associated with relabelling  $v$  is at most  $2V$ .
- Total increase associated with all relabellings is at most  $2V^2$ .

**Saturating Pushes**

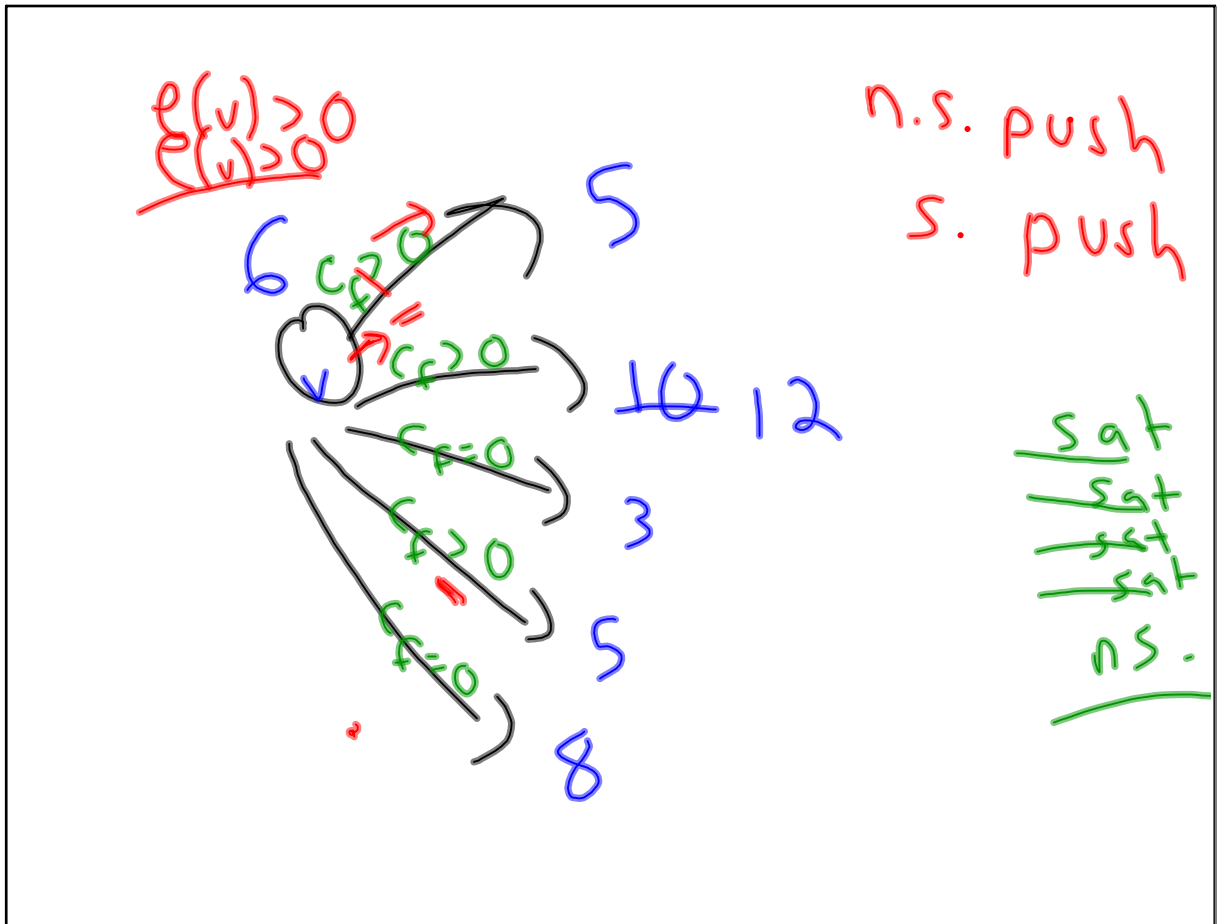
- A saturating push leaves excess at  $v$ . It adds excess to  $w$ .
  - If  $w$  already had excess, then  $\Phi$  is unchanged.
  - If  $w$  did not have excess, then  $\Phi$  increases by  $d(w)$ , which is at most  $2V$ .
- There are at most  $O(EV)$  saturating pushes, therefore total increase due to saturating pushes is  $O(EV^2)$ .

now  $e(w)x$

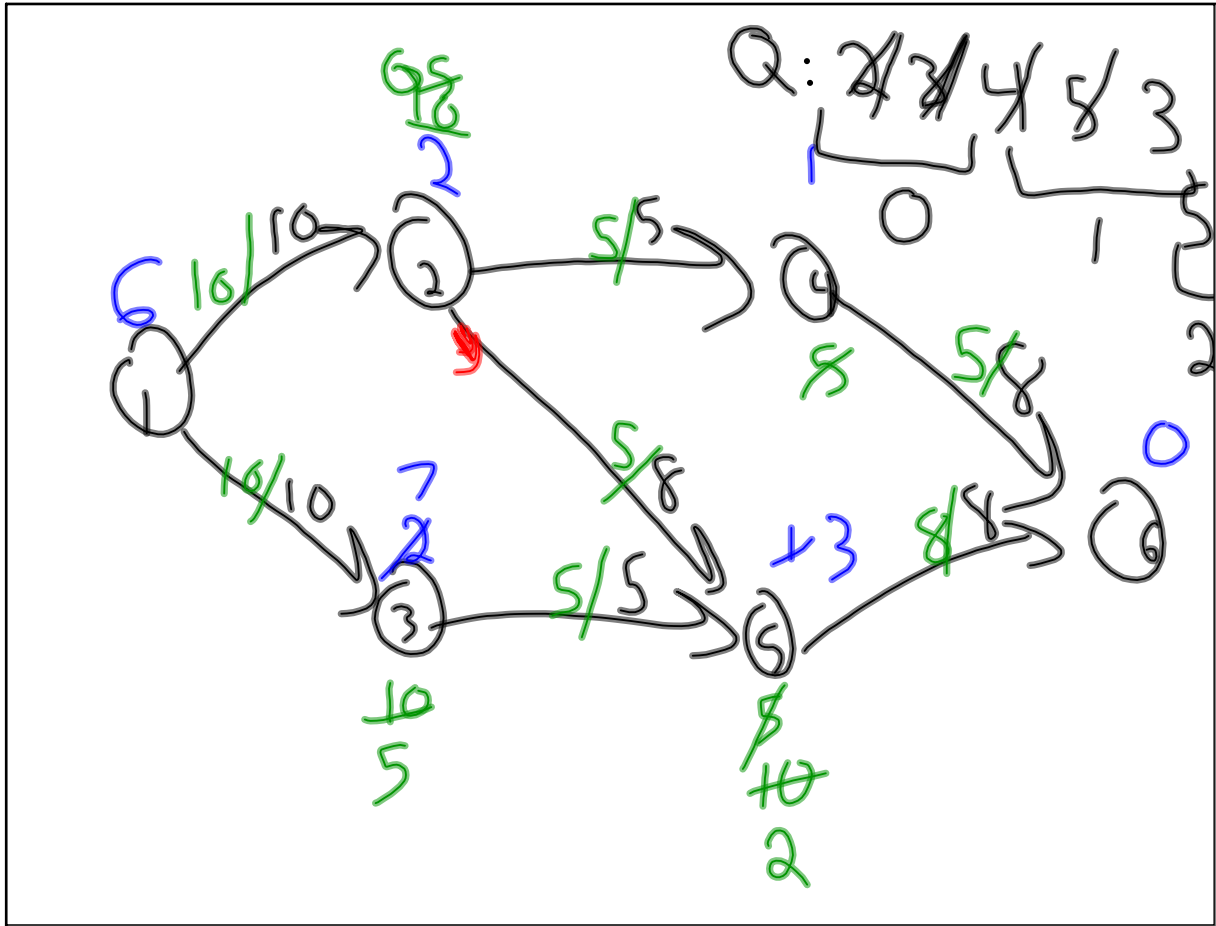
Mar 1-11:17 AM



Mar 1-11:18 AM



Mar 1-11:28 AM



Mar 1-11:41 AM

Look at  $Q$  during a phase

Case 1: Some vertex is relabelled during a phase

Case 2: No vertex is relabelled during a phase.

Mar 1-11:53 AM

Case 1:  $\Phi$  can increase, decrease, or stay same.

At most  $O(n^2)$  relabellings,  $\therefore$  at most  $O(n^2)$  case 1 phases.

Total increase in  $\Phi$  due to relabellings is at most  $O(n^2)$

Mar 1-11:55 AM

Case 2: No relabelling occurs

$abcd \mid xy?c$

If  $v^*$  had  $\max d(v)$  then  $v^*$  is not in  $Q$  at end of phase. All new vertices added to  $Q$  have  $d(w) < d(v^*)$   
 $\Rightarrow \Phi$  decreases by at least 1.

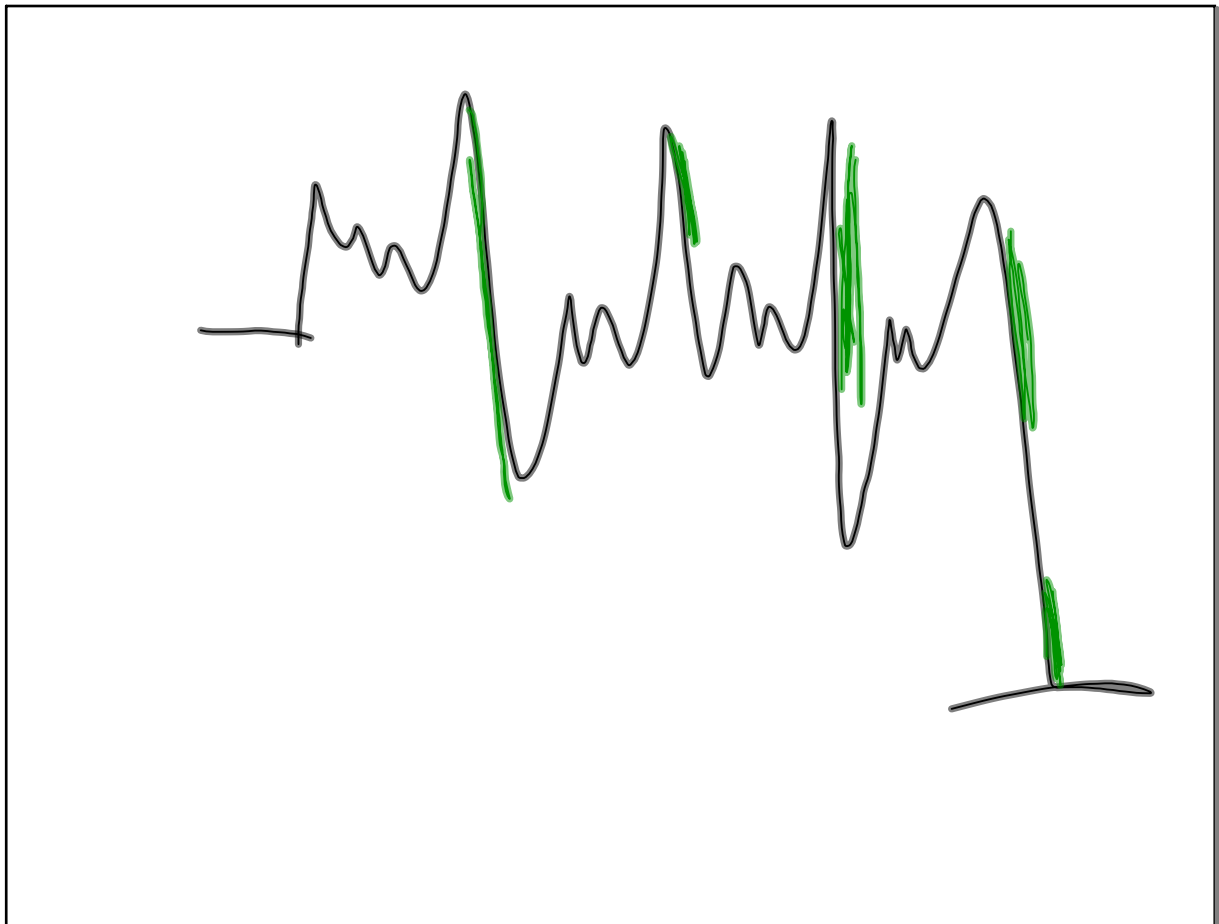
Mar 1-12:01 PM

Total increase in  $\Phi$  over the whole  
alg is  $\leq 2n^2$

$$\begin{aligned} \Phi_{\text{init}} &\leq n-1 & \text{total decrease} \\ & & \leq 2n^2 + n-1 \\ \Phi_{\text{final}} &= 0 & = O(n^2) \end{aligned}$$

Total # of case 2 phases  $= O(n^2)$

Mar 1-12:07 PM



Mar 1-12:12 PM