LP Max Flow

max $\sum_v f(s,v)$

s.t.

$y(v,w)$ $\quad f(v,w) \le c(v,w) \quad \forall (v,w)\in E$

$z(v)$ $\quad -\sum_w f(v,w) + \sum_w f(w,v) = 0 \quad \forall v \in V-\{s,t\}$

$f(v,w) \ge 0$

max $c^T x$ s.t. $Ax \le b$, $x \ge 0$ $\qquad$ min $b^T y$ s.t. $A^T y \ge c$, $y \ge 0$

min $\sum_{(v,w)\in E} c(v,w)\,y(v,w) + \cancel{\sum_v 0\, z(v)}$

s.t.

$z(T)=0$, $z(S)=1$

$y(v,w) - z(v) + z(w) \ge 0 \qquad (v,w)\in E,\ v \ne s,\ w \ne t$

$y(s,w) + z(w) \ge 1 \qquad (s,w) \in E$

$y(v,t) - z(v) \ge 0 \qquad (v,t) \in E$

$y(s,t) \ge 1 \qquad (s,t) \in E$

$y(v,w) \ge 0$, $z(v)$ unrestricted

$z(s)=1$, $z(t)=0$

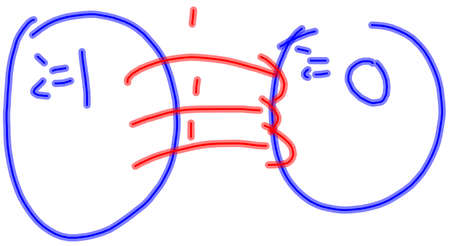
$v\in S,\ w \in T$: $y(v,w) - 1 + 0 \ge 0$, $y(v,w) \ge 1$

$v \in T,\ w \in S$: $y(v,w) - 0 + 1 \ge 0$, $y(v,w) \ge -1$

$y(v,w) \ge 0$ same side

Mar 6-11:00 AM

Any cut is a feasible sol'n to dual. Min cut is a fea. sol'n to dual.



OPT dual $\le$ min-cut

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Claim min-cut ≤ OPT dual.

Pf Let y^*, z^* be a feas. dual sol'n w/ $z^*(s)=1$

Pick $\delta \in (0,1]$ unif at random.

$$S = \{ v \in V \mid z^*(v) \ge \delta \}$$

Defines a cut.

$v \to w$, $z(v) \ge \delta$, $z(w) < \delta$

$$\Pr[(v,w) \text{ crosses cut}] = \Pr(z^*(w) < \delta \le z^*(v)) = \max(z^*(v) - z^*(w), 0) \le y^*(v,w)$$

$$E(\text{capacity of cut defined by } S) = \sum_{(v,w)\in E} c(v,w) \cdot \Pr[(v,w) \text{ crosses cut}] \le \sum_{(v,w)\in E} c(v,w)\, y^*(v,w).$$

$\Rightarrow \exists$ a cut of value $\le \sum c(v,w)\, y^*(v,w)$

min cut $\le$ Dual OPT.

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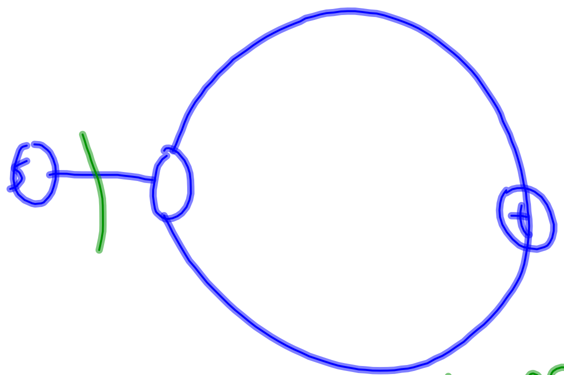

value of flow

$(\lg(nC))$

If there is an alg that know value of flow and runs in time X, then there is an alg. that does not the value of flow & runs in time $O(X \lg X)$.

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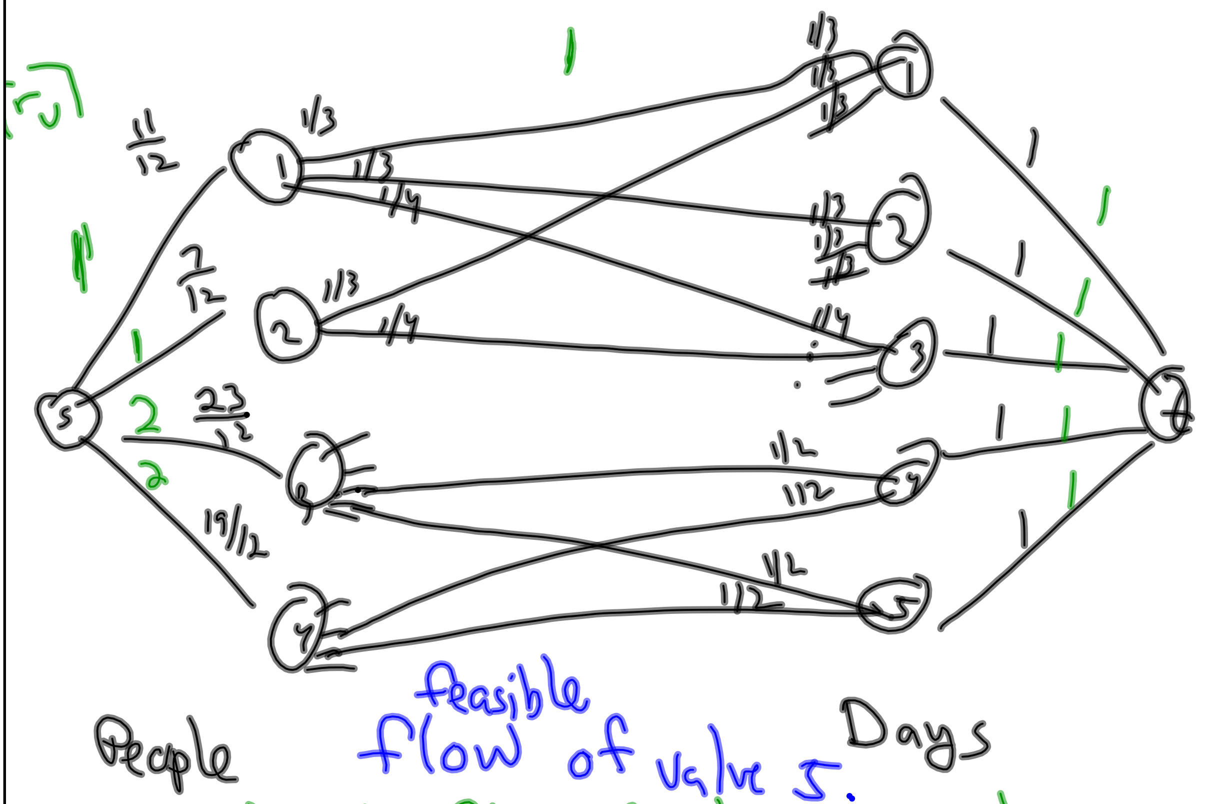

feasible flow of value 5.

feasible integral flow of value 5 exists.

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wins (Toronto + Yankess)

$\geq 88 + 93 + 6 = 187$

$\max(T, Y) \geq \left\lceil \frac{187}{2} \right\rceil = 94$

$\therefore$ Boston is eliminated.

Mar 6-12:13 PM