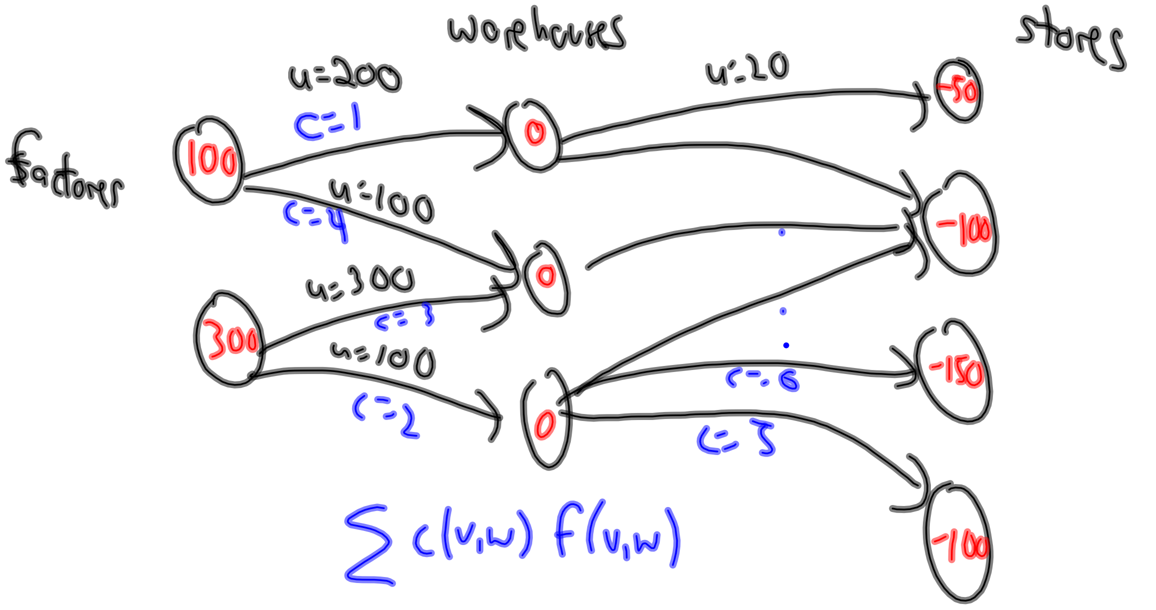

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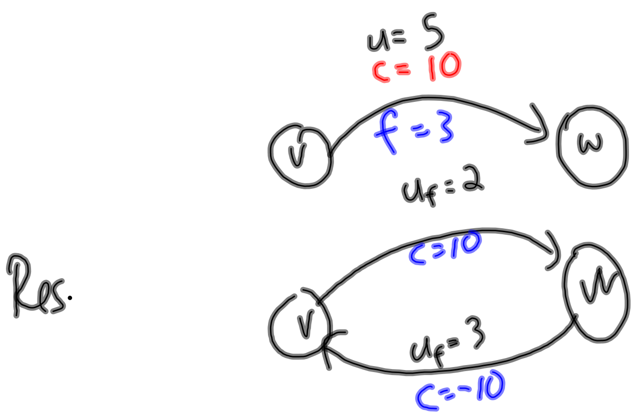
Residual Graph

Residual

- Capacity is as for flow (now use $u_f(v,w)$ for residual capacity
- If $(v,w) \in E$ and $(w,v) \in E_f$ then $c(w,v) = -c(v,w)$.



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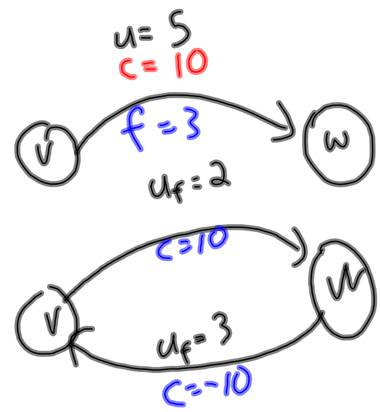
Residual Graph

Residual

- Capacity is as for flow (now use $u_f(v,w)$ for residual capacity
- If $(v,w) \in E$ and $(w,v) \in E_f$ then $c(w,v) = -c(v,w)$.

Res.



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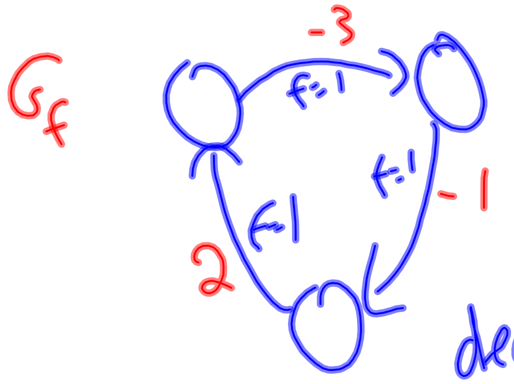
f optimal $\Leftrightarrow$ G_f no neg. cost cycles

Pf

$\Rightarrow$ if G_f has a neg. cost cycle then f is not optimal.



costs

$u_f > 0$

$-3 \cdot 1 + -1 \cdot 1 + 2 \cdot 1$
$= -2$

decreased the flow by 2

$\Rightarrow$ f was not optimal $\square$

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If a G_f has no neg cycles, then f is optimal.

Think about f & f^*, the opt. flow.

$f - f^*$ can be decomposed into a collection of cycles, in G_f

if G_f has no neg cycles, then

$f - f^*$ is a collection of non-neg. cycles $\Rightarrow c(f) \leq c(f^*)$

$\Rightarrow$ f is optimal.

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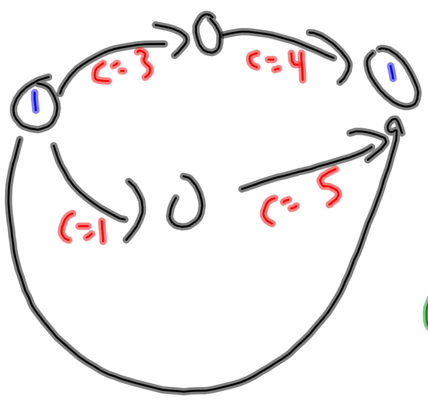
difference between two feas. mcf is a collection of cycles

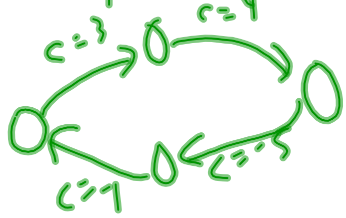
f_1 = top path

f_2 = bottom path



$f_1 - f_2$

cost = 1



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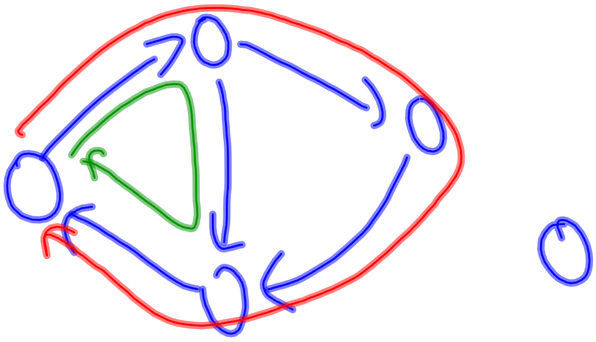
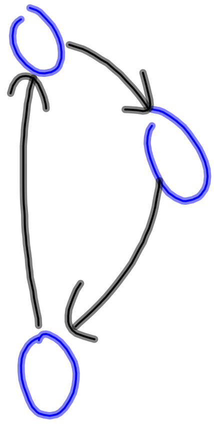
red - green = black





start w green
augment w/ black flow
I get the red flow.

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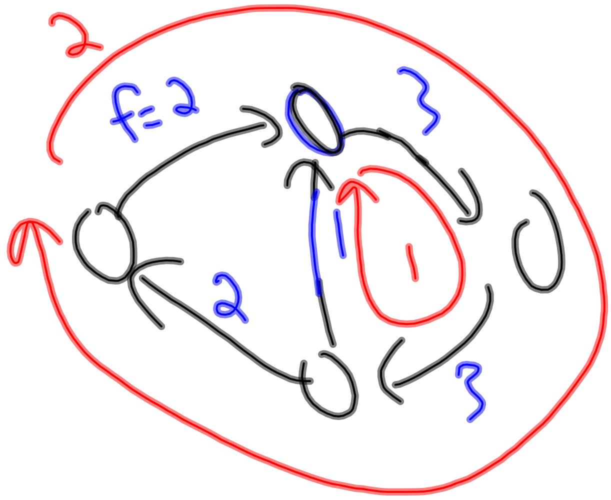

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Optimality 2: Reduced Cost Optimality

Reduced Cost Optimality: A feasible flow f is optimal iff there exsits potentials π such that

$$c^\pi(v,w) \geq 0 \quad \forall (v,w) \in G_f$$

Pf Show G_f has no neg cost cycles $\Leftrightarrow$

($\Leftarrow$) $c^\pi(v,w) \geq 0 \quad \forall (v,w) \in G_f$

$\Rightarrow$ for any cycle X, $\sum_{(v,w)\in X} c^\pi(v,w) \geq 0$

$\Rightarrow \sum_{(v,w)\in X} c(v,w) \geq 0.$

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Optimality 2: Reduced Cost Optimality

G_f no neg. cost cycles $\Rightarrow$

Reduced Cost Optimality: ~~A feasible flow f is optimal~~ iff there exsits potentials π such that

$$c^\pi(v,w) \geq 0 \quad \forall (v,w) \in G_f$$

$\Rightarrow$ Pick a source s, compute s.p. dists $d(v)$ from s

Set $\pi(v) = -d(v)$

$\left(d(w) \leq d(v) + c(v,w) \quad \forall (v,w) \in E \right)$

$c^\pi(v,w) = c(v,w) - \pi(v) + \pi(w)$

$= c(v,w) + d(v) - d(w) \geq 0$ $\square$

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Optimality 3: Complimentary Slackness

A feasible flow f is optimal iff there exsits potentials π such that for all edges $(v, w) \in G$

- if $c^{\pi}(v, w) > 0$ then $f(v, w) = 0$
- if $0 < f(v, w) < u(v, w)$ then $c^{\pi}(v, w) = 0$
- if $c^{\pi}(v, w) < 0$ then $f(v, w) = u(v, w)$.

$c(v,w) = c(w,v) = 0$

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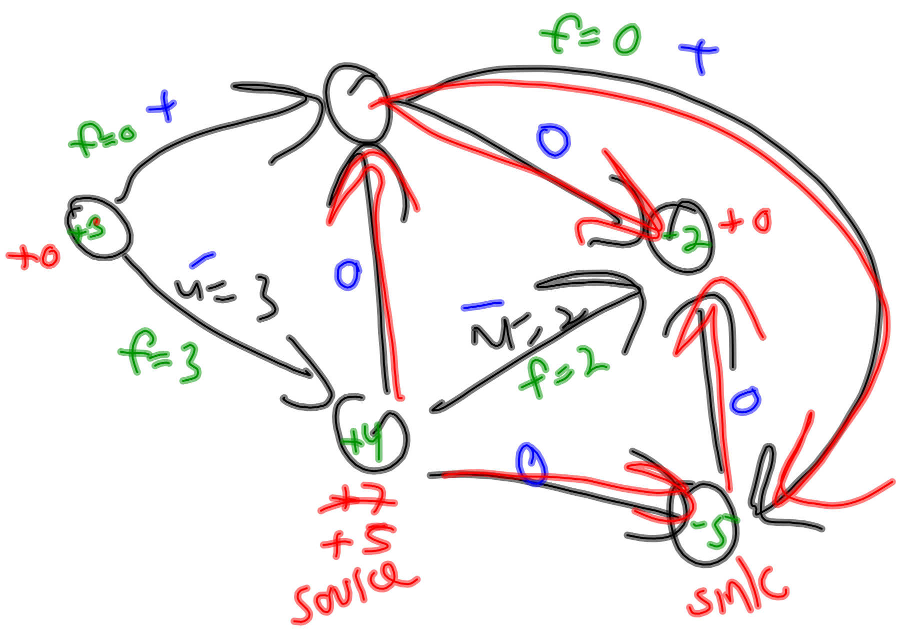
c^{π}



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