mcf.pdf - Adobe Reader

More analysis

Lemma: Let f be a feasible non-optimal flow. Let X be the minimum mean cycle in G_f . Then there exist π s.t.

$$c^{\pi}(v,w) = \mu(f) = -\epsilon(f) \ \ \forall (v,w) \in X$$

5

-1

$\mu = -1$

$c'(v,w) = c(v,w) + 1$

Mar 29-11:15 AM

mcf.pdf - Adobe Reader

Progress

Lemma: Let f be a feasible non-optimal flow. Let X be the minimum mean cycle in G_f . Suppose we push flow around X to obtain f' . Then $\epsilon(f') \le \epsilon(f) = \epsilon$

edge(s) leave G_f. That cannot increase ϵ

$c^{\pi}(v,w) = -c^{\pi}(w,v)$

$\epsilon(f')$

Any new cycle in $G_{f'}$ has at least one edge w/ $c^{\pi} = \epsilon$ and all others $\geq -\epsilon$. $\therefore$ mean is $> -\epsilon$.

Mar 29-11:42 AM

mcf.pdf - Adobe Reader

Summary

- In m iterations, ϵ decreases by a $1 - 1/n$ factor.
- In nm iterations, ϵ decreases by a $(1 - 1/n)^n \approx 1/e$ factor.
- Initially $\epsilon \leq C$
- We stop when $\epsilon \leq 1/n$
- Decrease by a factor of e $\ln(nC)$ times.
- Therefore, number of iterations is $O(nm \log(nC))$
- Running time is $O(n^2m^2 \log(nC))$

$2^{32}-1$, $2^{32}-1$, $2^{32}-1$, $2^{32}-1$

Nice feature of algorithm: No explicit scaling. Eplicit scaling enforces a lower bound.

lg C added a bit to the capacity

Mar 29-12:07 PM

mcf.pdf - Adobe Reader

Strongly Polynomial Algorithm

- Recall that strongly polynomial means polynomials in n and m and "independent" of C and U.
- We have seen strongly polynomial algorithms for maximum flow.
- No strongly polynomial algorithm is known for linear programming.
- No strongly polynomial algorithm is known for multicommodity flow.
- We will see a strongly polynomial algorithm for minimum cost flow, one of the "hardest" problems for which such an algorithm exists.
- Strongly polynomial is mainly a theoretical issue.

Theorem: The minimum mean cycle algorithm runs in $O(n^2m^3 \log n)$ time.

s.p. min cost flow multi LP

strong poly don't know

Mar 29-12:13 PM

mcf.pdf - Adobe Reader

File Edit View Window Help

23 / 28 76% Tools Comment

Show next page (Right Arrow)

Measured Progress

Lemma: Let f be a feasible non-optimal flow. Suppose that we execute m iterations of the minimum-mean cycle algorithm to obtain f'. Then, if the algorithm has not terminated, we have that

$$\epsilon(f') \le \left(1 - \frac{1}{n}\right)\epsilon(f)$$

Pf Fix π from f

Case 1) each iteration cancels a cycle X w/ $c^\pi(v,w) \le 0$ $\forall (v,w) \in X$.

Each iteration removes at least one edge w/ $c^\pi(v,w) \le 0$ from res. graph, and might insert edges w/ $c^\pi(v,w) \ge 0$ in the res. graph.

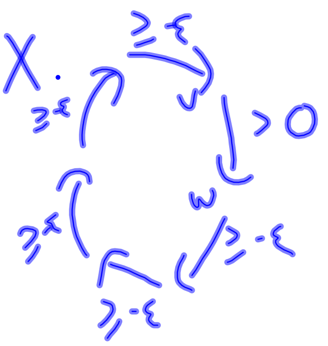
After m iterations, flow f' and in $G_{f'}$ $c^\pi(v,w) \ge 0$ $\forall (v,w) \in G_{f'}$!

$\Rightarrow f'$ is optimal.

Mar 29-11:45 AM

Case 2: Some iteration cancels a cycle X w/ $c^\pi(v,w) > 0$ for some $(v,w) \in X$.

Look at X.



$-\epsilon$
$-\epsilon$
$-\epsilon$
$-\epsilon(1-\frac{1}{n})$

This is the min mean cycle.

$$\mu(f) \ge \frac{-\epsilon(|X|-1)+0}{|X|} \ge \frac{-\epsilon(n-1)}{n} = -\epsilon\left(1-\frac{1}{n}\right)$$

Let f' be the flow after this iteration

$$\epsilon'(f) \le \epsilon\left(1-\frac{1}{n}\right)$$

Mar 29-11:52 AM