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Analysis

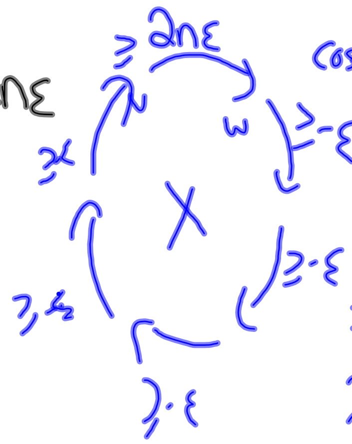
Theorem If $|c^\pi(v,w)| \ge 2n\epsilon(f)|$, then (v,w) is ϵ-fixed.

$\varepsilon = \varepsilon(f)$

Pf 1) $c^\pi(v,w) \geq 2n\varepsilon$

f.p.o.c. Suppose I push flow on (v,w)

The cycle containing (v,w) has >0 cost.



$\text{cost}^\pi(X) \geq 2n\varepsilon + (n-1)(-\varepsilon) = n\varepsilon + \varepsilon > 0.$

Apr 3-11:07 AM

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Analysis

Theorem If $|c^\pi(v,w)| \ge 2n\epsilon(f)|$, then (v,w) is ϵ-fixed.

2) $c^\pi(v,w) \leq -2n\varepsilon$

(v,w) not in G_f. For (v,w) to enter res. graph, we have to push flow on (w,v)

but $c^\pi(w,v) = -c^\pi(v,w) \geq 2n\varepsilon$, by 1 we don't push flow on (w,v)

Apr 3-11:21 AM

$nm(\ln n+1)$ iterations

f, π, ε

$nm(\ln n + 1)$ its.

f, π', ε'

$$\varepsilon' \leq \frac{\varepsilon}{e^{(\ln n+1)}} = \frac{\varepsilon}{e\, n} \leq \frac{\varepsilon}{2n}$$

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Consider X a cycle cancelled in the first iteration

$\mu(X) = -\varepsilon$

$c^{\pi}(v,w) = -\varepsilon \quad \forall (v,w) \in X$

$$\sum_{(v,w)\in X} c^{\pi}(v,w) = -\varepsilon |X|$$

$$\Rightarrow \sum_{(v,w)\in X} c^{\pi'}(v,w) = -\varepsilon |X|$$

$$\Rightarrow \frac{\sum_{(v,w)\in X} c^{\pi'}(v,w)}{|X|} = -\varepsilon$$

some edge (v,w) has $c^{\pi'}(v,w) \geq -\varepsilon$

$(\varepsilon' \leq \varepsilon/2n)$ $\geq -2n\varepsilon'$

$(-\varepsilon \geq -2n\varepsilon')$

$\Rightarrow (v,w)$ is ε' fixed.

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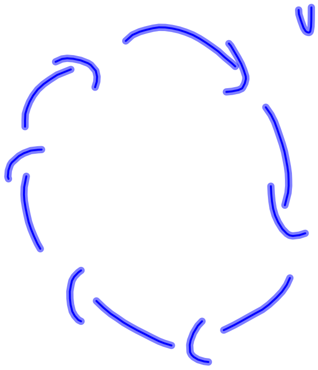

v. sig negative cycle w/r/t ε'

$|X|(-\varepsilon)$

ε'

$\varepsilon' = 1$

$\varepsilon = 10^6$

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$c'(v,w) = c(v,w) - p(f)$

$d(v)$: $c'(v,w)$

$\pi(v) = -d(v)$

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~~plane~~ train holds p people

b_{ij} = # passengers who want to fly from i to j $i<j$

$$1 \to 2 \to \circ \to \cdots \to n$$

m_{ij} - payment by a passenger going from i to j

$m_{ij} \neq m_{ik} + m_{kj}$

Accept a subset of requests to maximize profit

Apr 3-11:41 AM

ideas: f_{ij} people traveling i to j

$-m_{ij} f_{ij}$ payment

$$\overset{b_{13}}{\circ} \to \circ \xrightarrow[u=p]{} \overset{-b_{13}}{\circ} \to \cdots \to \circ \to \circ$$

difficulties: · edge costs different amts. for different people

· deal w/ unmet demand

need: path for not flying

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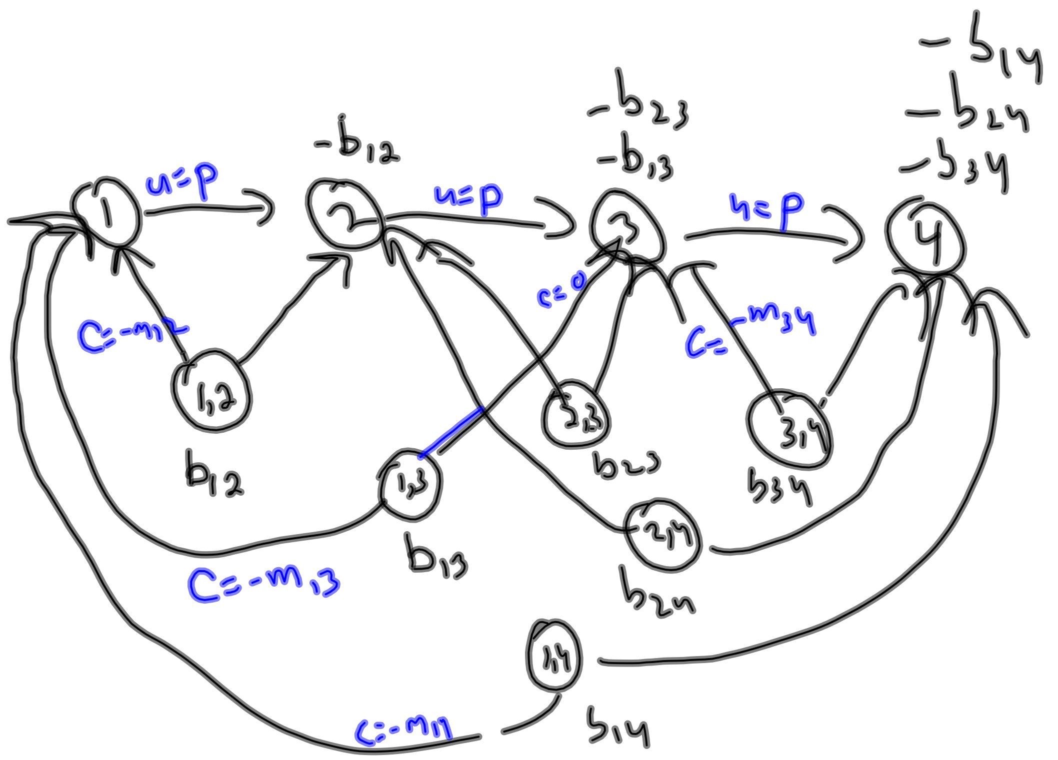

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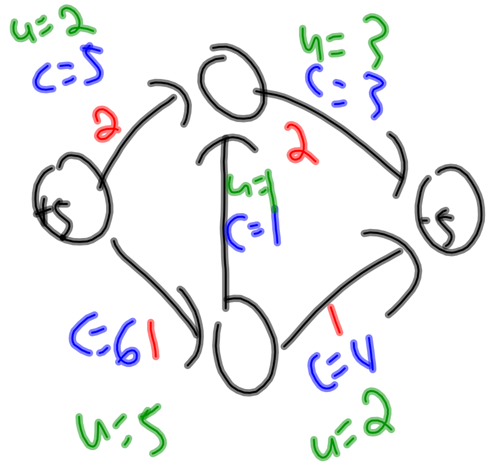
Another approach to min cost flow.



1 unit of flow
send on shortest path.
(w.r.t c)

Repeat. Send flow along shortest path in Gf.

Apr 3-12:01 PM

Q: What if s.p. are not defined.

Pseudoflow:

$$0 \le f(v,w) \le u(v,w) \quad \forall (v,w) \in E.$$

Define: $e(v) = b(v) + \sum_w f(w,v) - \sum_w f(v,w)$.

excess

1) if $e(v) = 0$ $\forall v \in V$ then a pseudoflow is a flow.

2) reduced cost opt. of a pseudoflow $c^\pi(v,w) \ge 0$ $\forall (v,w) \in G_f$.

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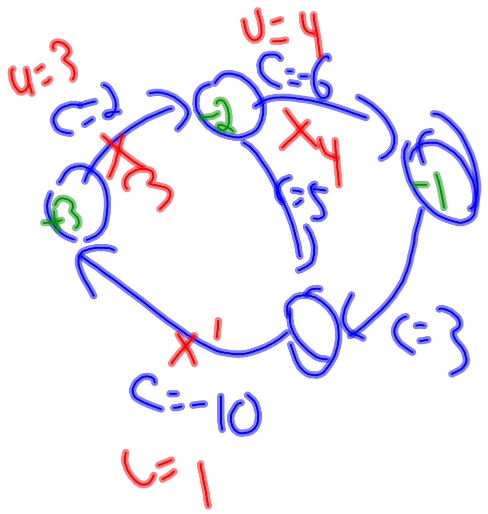

Remove neg. cost cycles:

Saturate all neg. cost edges.

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