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Capacity Scaling Algorithm for Minimum Cost Flow

Successive Shortest Path

$f = 0$; $\pi = 0$
$e(v) = b(v)\ \forall v \in V$
$\Delta = 2^{\lfloor U \rfloor}$
while $\Delta \geq 1$
    (Δ scaling phase)
    for every edge $(v, w) \in G_f$
        if $u_f(v, w) \geq \Delta$ and $c^\pi(v, w) < 0$
            Send $u_f(v, w)$ units of flow on (v, w); update f, e
    $S(\Delta) = \{v \in V : e(v) \geq \Delta\}$
    $T(\Delta) = \{v \in V : e(v) \leq -\Delta\}$
    while $S(\Delta) \neq 0$ and $T(\Delta) \neq 0$
        Pick a node $k \in S(\Delta)$ and $\ell \in T(\Delta)$
        Compute $d(v)$, shortest path distances from k in $G_f(\Delta)$ w.r.t. edge distances c^π.
        Let P be a shortest path from k to ℓ.
        Set $\pi = \pi - d$
        Let $\delta = \min\{e(k), -e(\ell), \min\{u_f(v, w) : (v, w) \in P\}\}$
        Send δ units of flow on the path P
        Update f, $G_f(\Delta)$, $S(\Delta)$, $T(\Delta)$ and c^π.
    $\Delta = \Delta/2$

cap $= \cancel{\Delta - 1}$ $2\Delta - 1$

$-$cost

$U_f(v,w) = u(v,w) - f(v,w) + f(w,v)$

$\delta \geq \Delta$

Δ

$f(v,w)$ mult of Δ

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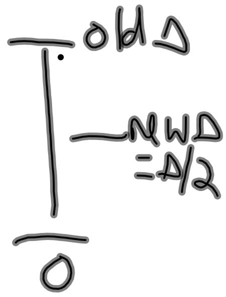
$$|S(\Delta)| = \sum_{v \in S(\Delta)} e(v)$$

$$|T(\Delta)| = -\sum_{v \in T(\Delta)} e(v)$$



assume we end iteration because $S(\Delta) = \emptyset$

set $\Delta = \Delta/2$

new $S(\Delta)$

$$|S(\Delta)| = \sum_{v \in S(\Delta)} e(v) \leq \sum_{v \in S(\Delta)} 2\Delta$$

$$\leq 2n\Delta.$$

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Capacity Scaling Algorithm for Minimum Cost Flow

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1 $f = 0;\ \pi = 0$
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3 $\Delta = 2^{\lfloor U \rfloor}$
4 while $\Delta \geq 1$
5 (Δ scaling phase)
6 for every edge $(v,w) \in G_f$
7 if $u_f(v,w) \geq \Delta$ and $c^\pi(v,w) < 0$
8 Send $u_f(v,w)$ units of flow on (v,w); update f, e
9 $S(\Delta) = \{v \in V : e(v) \geq \Delta\}$
10 $T(\Delta) = \{v \in V : e(v) \leq -\Delta\}$
11 while $S(\Delta) \neq 0$ and $T(\Delta) \neq 0$
12 Pick a node $k \in S(\Delta)$ and $\ell \in T(\Delta)$
13 Compute $d(v)$, shortest path distances from k in $G_f(\Delta)$ w.r.t. edge distances c^π.
14 Let P be a shortest path from k to ℓ.
15 Set $\pi = \pi - d$
16 Let $\delta = \min\{e(k), -e(\ell), \min\{u_f(v,w) : (v,w) \in P\}\}$
17 Send δ units of flow on the path P
18 Update f, $G_f(\Delta)$, $S(\Delta)$, $T(\Delta)$ and c^π.

$|S(\Delta)| \leq 2n\Delta$

$2m\Delta$

w $e(w)\uparrow$

$u_f(v,w) < 2\Delta$

no neg red. cost edges in $G_f(\Delta)$

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Hit while loop in line 11

$$|S(\Delta)| \leq 2n\Delta + 2m\Delta$$

$S(\Delta)$ v $\xrightarrow{\text{send at least } \Delta}$ w $T(\Delta)$

$e(v)$ decreases by $\geq \Delta$

$e(w)$ inc. by $\geq \Delta$

each push decreases $|S(\Delta)|$ by $\geq \Delta$

$$\#\text{ iterations of while loop} \leq \frac{2n\Delta + 2m\Delta}{\Delta} = 2(n+m) = O(m).$$

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$O(mn \lg n \lg C)$ is possible

using cost scaling, subproblem is solving via a push/relabel type algorithm.

labels $\cong$ π values.

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matching.pdf - Adobe Reader

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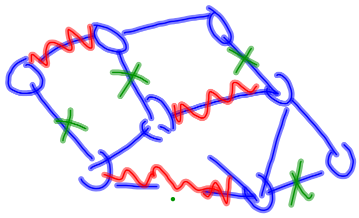
Matchings

Definition: Give an undirected graph G, a matching M is a subset of the edges $E \subseteq M$ such that each vertex $v \in V$ is incident to at most one edge from M.



Variants of Matching

- Graph can be bipartite or general
- Graph can be weighted or unweighted

Terms

- A matching M such that, for all edges $e \notin M$, $M \cup \{e\}$ is not a matching, is called maximal.
- A maximum cardinality matching is called maximum.
- A matching of size $|V|/2$ is called perfect.
- The weight of a matching M is $w(M) = \Sigma_{e \in M} w(e)$.
- A maximum weight matching is the matching of maximum weight.
- All variants polynomial time, bipartite matching seems "easier".

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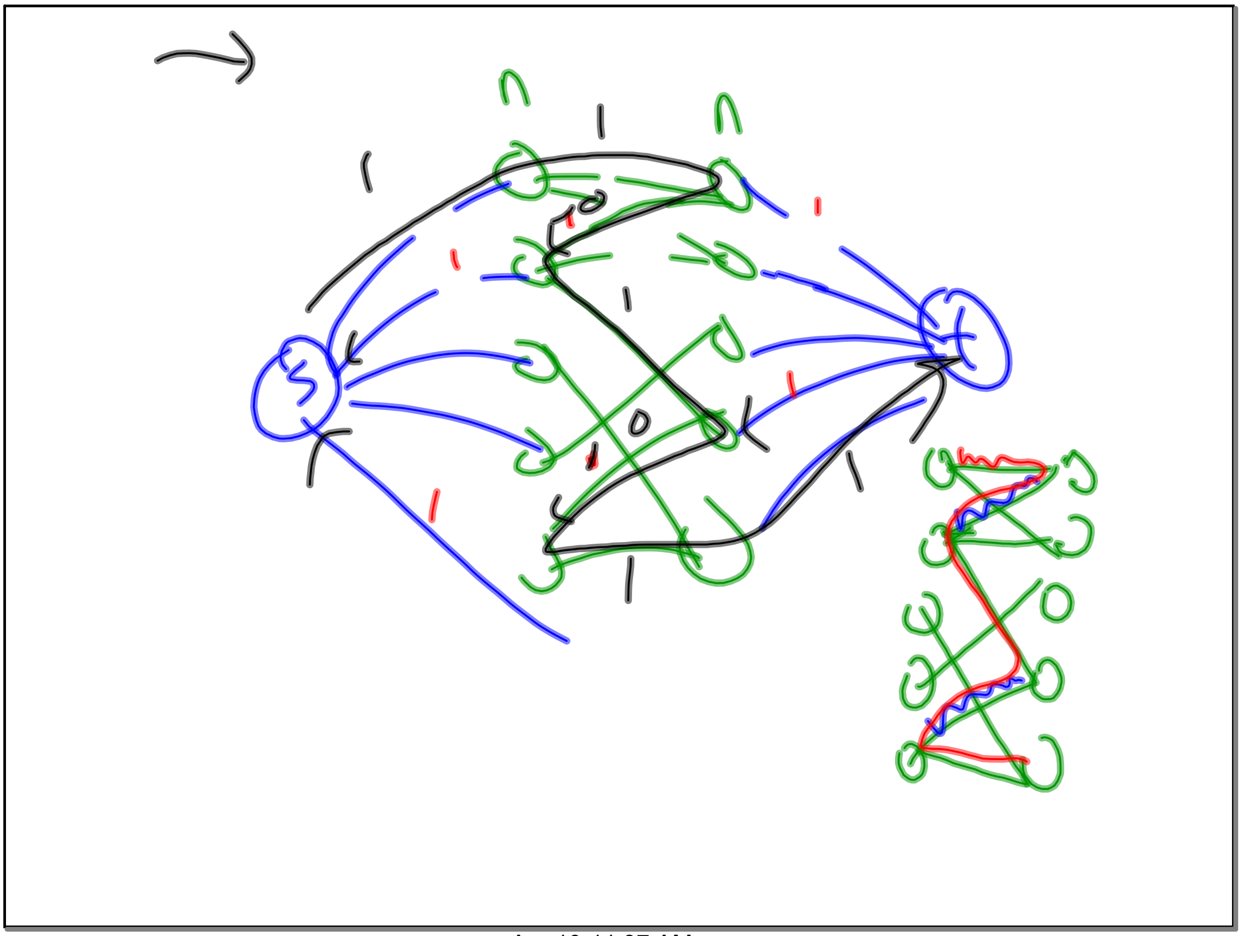

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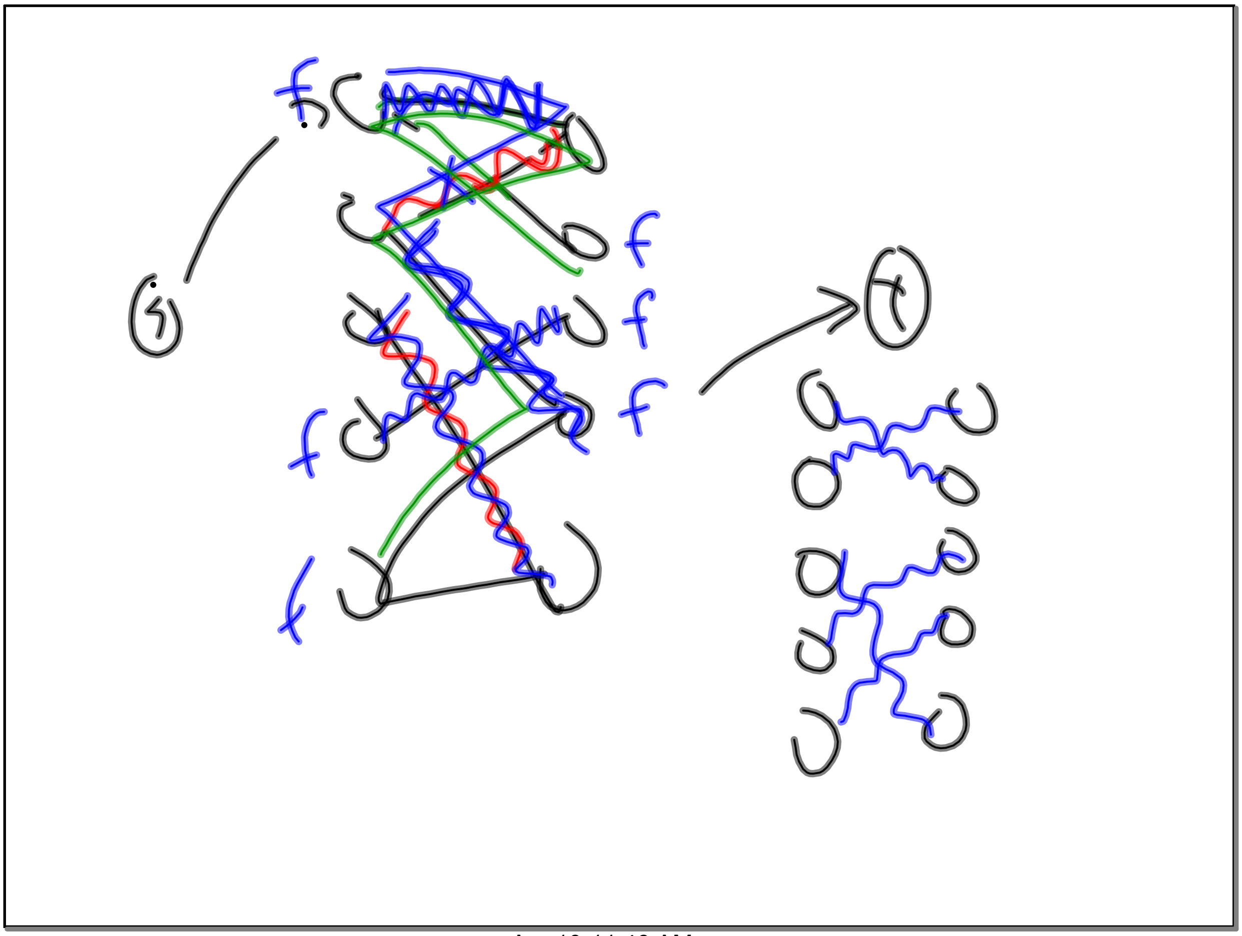

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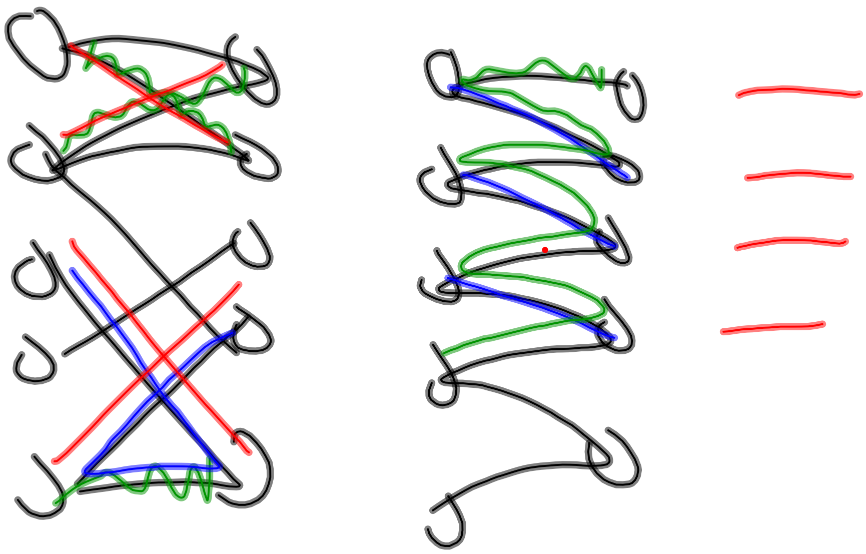

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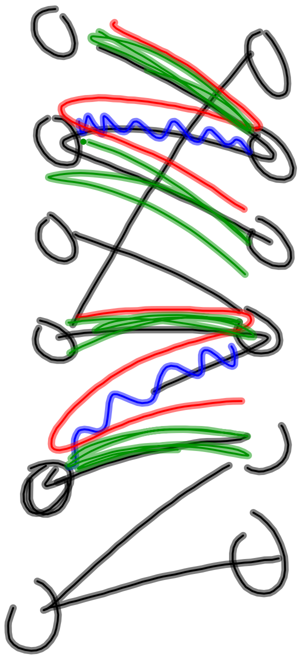

2 v.d. aug paths

$|M| = 2$

2 v.d. aug. paths

$|\text{new } M| = 4$

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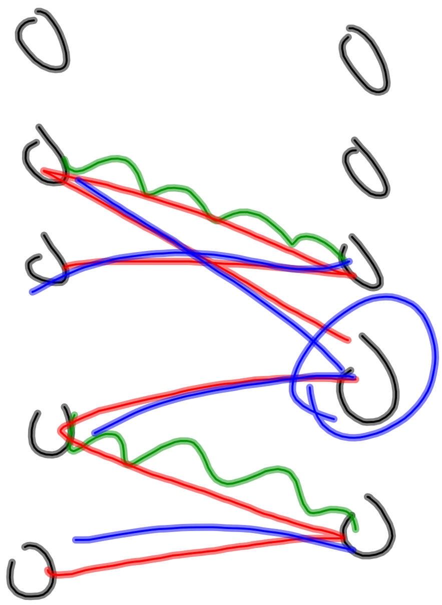

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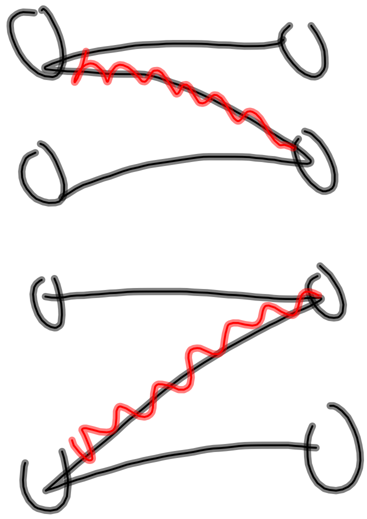

red is a maximal set of vertex disj. shortest aug. paths.

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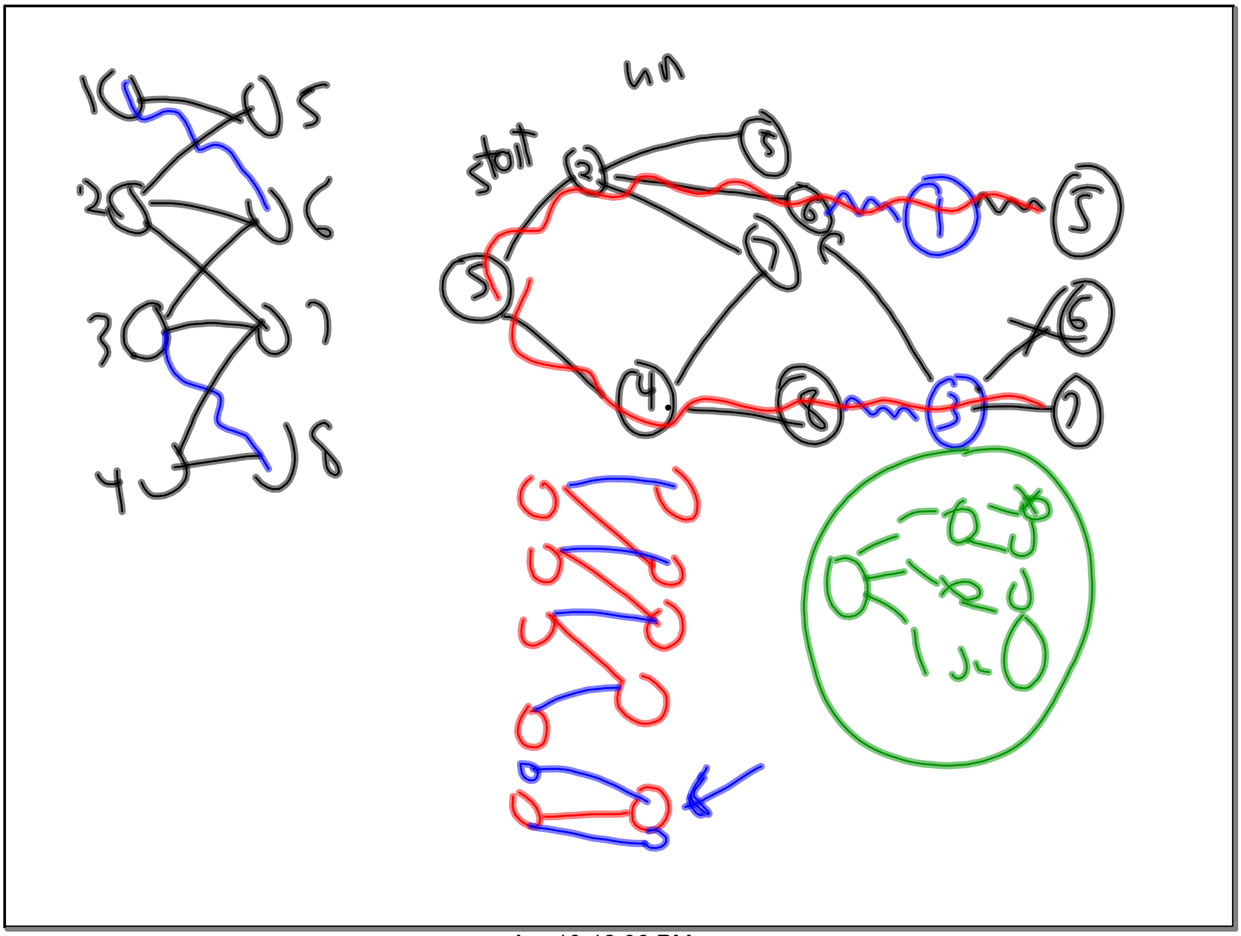

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36/60 avg
med.

2	2	0	4	6	5	1	1	1
20	25	30	35	40	45	50	55	60

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