matching.pdf - Adobe Reader

Main Lemma

Lemma Let ℓ be the length of a shortest augmenting path with respect to M. Let $P_1, \ldots, P_k$ be a maximal set of vertex disjiont shortest augmenting paths. Let $M' = M \oplus (P_1 \cup \ldots P_k)$. Let P be a shortest augmenting path with respect to M'. Then $|P| > \ell$.

$\cancel{M \oplus M'} = (P_1 \cup \cdots \cup P_k)$

$M \oplus M' \oplus P = (P_1 \cup \cdots \cup P_k) \oplus P$

$|(M' \oplus P)| = |M| + k + 1$

$M \oplus (M' \oplus P)$ must contain at least

$|M' \oplus P| - |M|$ v.d. aug. paths with M.

$|M| + k + 1 - |M| = k+1$

Shortest aug. path w.r.t. M has ℓ edges.

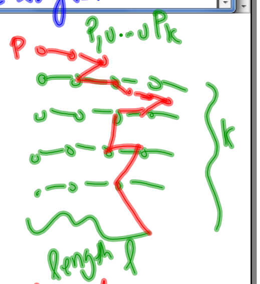
$|M \oplus (M' \oplus P)| \geq (k+1)\ell$.

$\Rightarrow |(P_1 \cup \cdots \cup P_k) \oplus P| \geq (k+1)\ell$

$(P \cap (P_1 \cup \cdots \cup P_k) \neq \emptyset)$

P has $\geq \ell$ edges not in $(P_1 \cup \cdots \cup P_k)$

has ≥ 1 edge in $(P_1 \cup \cdots \cup P_k)$ $\rightarrow \geq \ell + 1$ edges.



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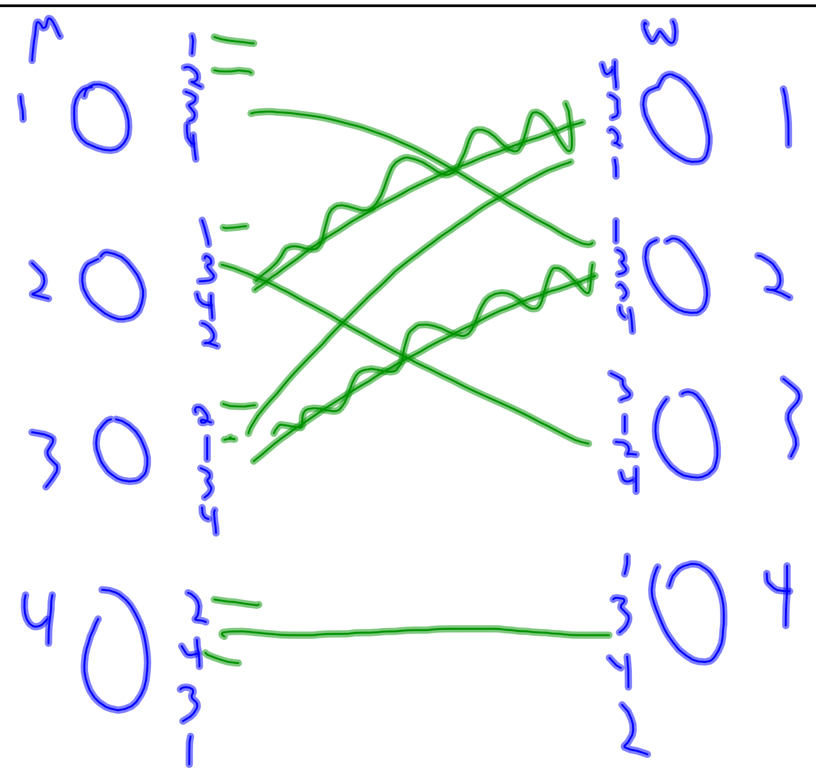

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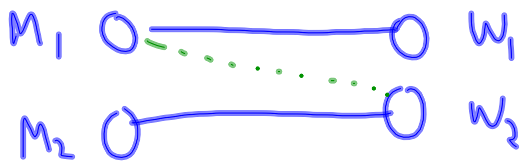
Matching is stable

Suppose unstable. $\exists$ a pair (unstable) (m_1, w_2)



m_1 prefers w_2 to w_1

w_2 prefers m_1 to m_2

m_1 has proposed to w_2, w_2 has rejected m_1 at that time

- w_2 rejected m_1 for someone, m_i, she preferred to m_1
- w_2 is currently married to m_2, who is either m_i or someone she prefers to m_i

$\Rightarrow$ w_2 does not prefer m_1 to m_2 X

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Shortest path

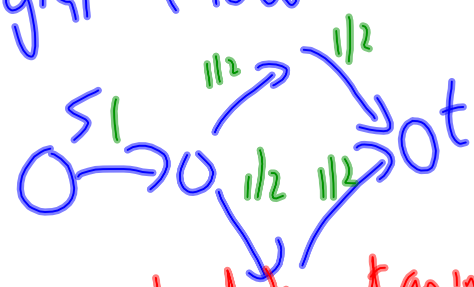
Max flow

Min Cost Flow

data integral

Found integral soln's.

$\exists$ opt. integral flow



Used simplex - guaranteed to get an integral soln.

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In general, to guarantee integer sol'ns, I need to solve an integer program.

Which is much slower than an LP.

(max, MC)
Flow problems have the property that LP basic feasible solutions are guaranteed to be integral.

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Max $\sum_{w} f(s,w)$
s.t.

$$f(v,w) \le u(v,w)$$

$$\sum f(v,w) - \sum f(w,v) = 0$$

$$f(v,w) \ge 0$$

$$\underline{\underline{A}} \cdot f \le b$$

$$\begin{array}{c} \\ v_1 \\ \vdots \\ v_n \end{array} \begin{array}{c} e_1 \; \cdots \; e_m \\ \begin{bmatrix} -1 & 1 & \\ +1 & -1 & \\ & +1 & \\ -1 & -1 & \end{bmatrix} \end{array} \begin{array}{c} f \\ \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix} \end{array}$$

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A matrix A is unimodular if every basis submatrix B of A has $\det(B) = \pm 1$

(Basis submatrix is a $p \times p$ submatrix w/ linearly ind. columns)

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