

basic feasible soln

x_B , there is an assoc. basis matrix B
s.t.

$$B x_B = b$$

Solve for x_B , by Cramer's rule, an

entry of x_B , $x_i = \frac{\det(\text{int. matrix})}{\det(B)}$

$$= \frac{\text{int}}{\pm 1} = \text{int.}$$

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nodes \rightarrow $\begin{bmatrix} +1 & -1 & \dots & +1 \\ -1 & +1 & \dots & +1 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -1 \end{bmatrix}$ $\cdot \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix} = 0$ flow cons.

Every sq. submatrix has det. 0, +1, -1.
Pf by induction:
 $k=1$ ✓
 $(k+1) \times (k+1)$

- 1) column w/ all 0's
- 2) each col has exactly 2 non-zeros
- 3) col. w/ one non-0

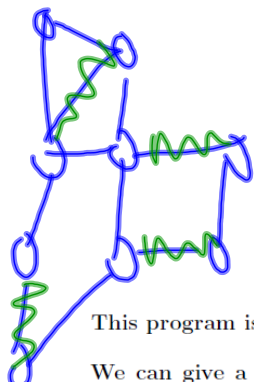
1) det = 0
3) $\begin{bmatrix} a & x \\ & k \times k \end{bmatrix}$
 $a \cdot \det(x)$
 $(-1) \times (-1 + 1 \ 0)$
 $= -1 + 1 \ 0$

2) $\begin{bmatrix} +1 & +1 & \dots & +1 \\ -1 & \dots & \dots & -1 \\ \dots & \dots & \dots & \dots \end{bmatrix}$
sum rows, I get 0
 \Rightarrow linear dependent
 \Rightarrow det = 0

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Non-bipartite matching

LP-relaxation



$$\max \sum_{(i,j) \in E} x_{ij} \quad (1)$$

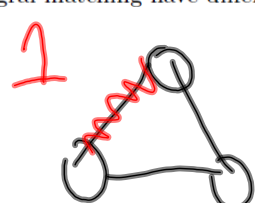
s.t.

$$\sum_{(i,j) \in E} x_{ij} \leq 1 \quad \forall i \in V \quad (2)$$
~~$$x_{ij} \in \{0, 1\} \quad (4)$$~~

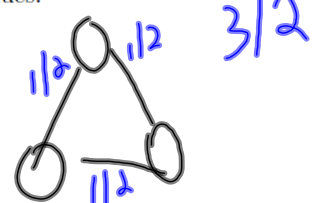
$$x_{ij} \geq 0 \quad (5)$$

This program is not totally unimodular.

We can give a graph for which the optimal fraction matching and the optimal integral matching have different values.



1



1/2 1/2 1/2 3/2

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a/c

[

node

...

I

...

a/c

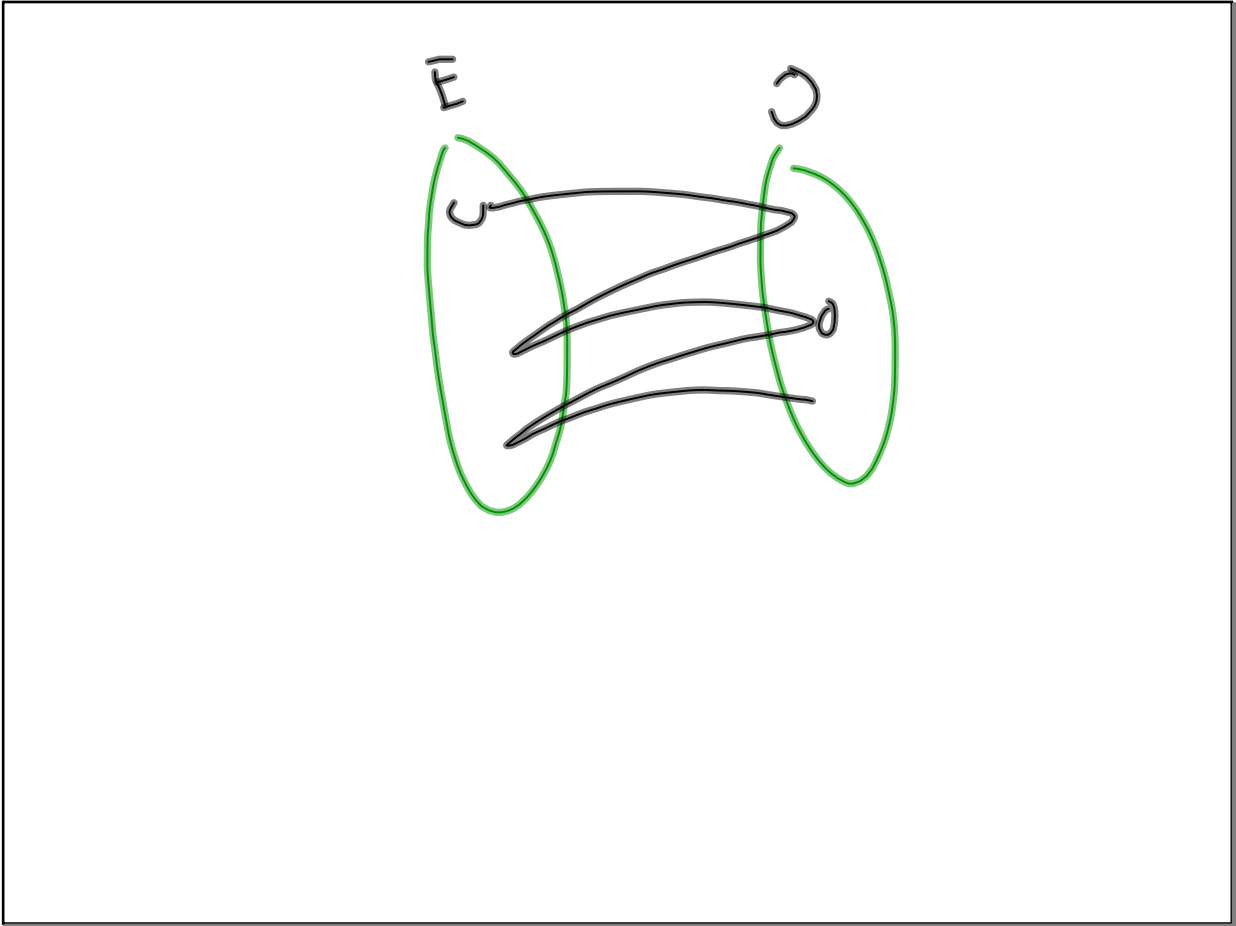
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cons.

cap.

still T.U.

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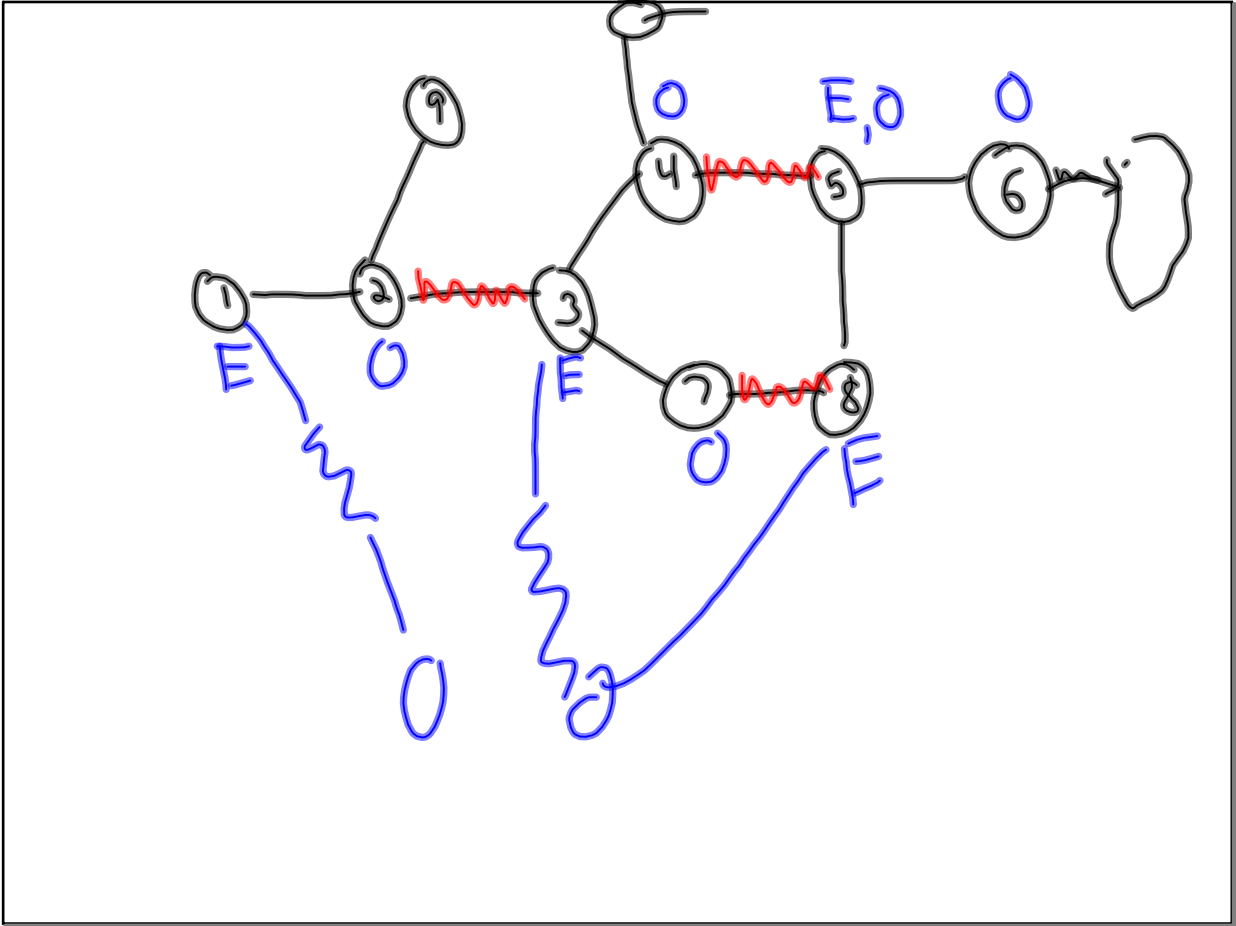
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P M M' opt. matching . If P is unmatched in M' , done
 $M \oplus M'$
 P not empty not in cycle
 Assume P is matched in M'

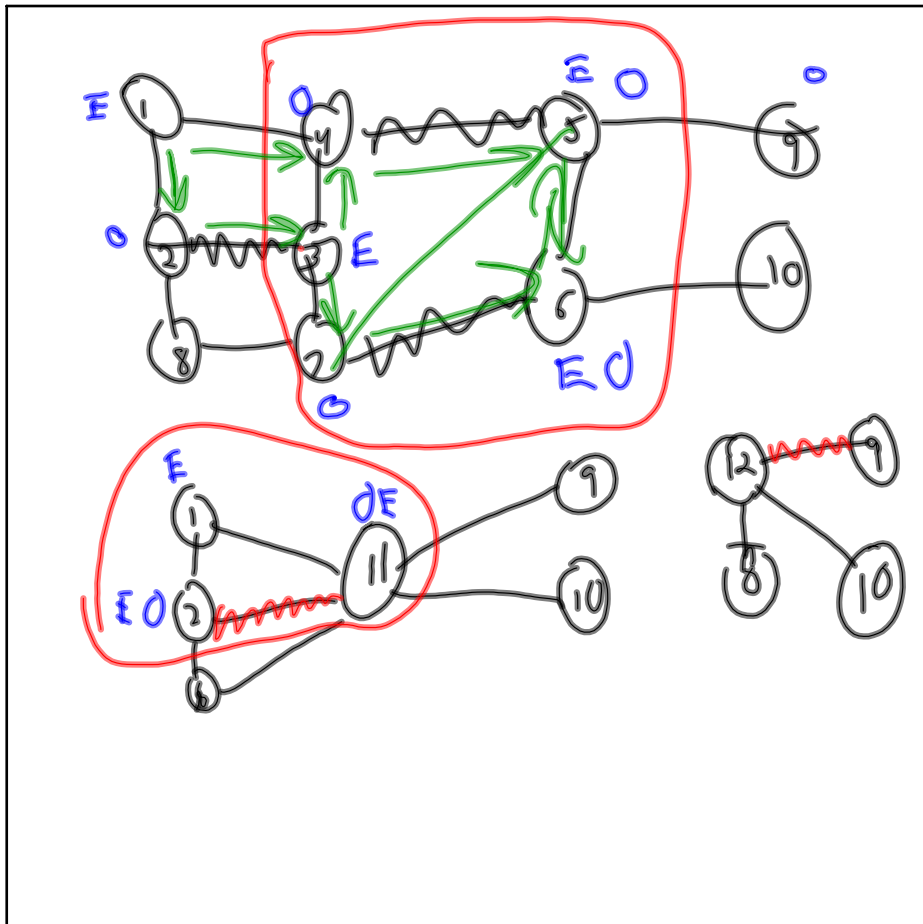
$P \xrightarrow{M'} \circ \xrightarrow{M} \circ \xrightarrow{M'} \circ \xrightarrow{M} \circ \xrightarrow{M'} \circ \xrightarrow{M} \circ$
 $P \xleftarrow{M'} \circ \xrightarrow{M} \circ \xrightarrow{M'} \circ \xrightarrow{M} \circ \xrightarrow{M'} \circ$

use the matching w/ M edges instead of M from the path.
 Aug path w/lt M .

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