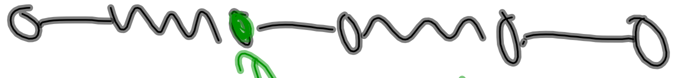
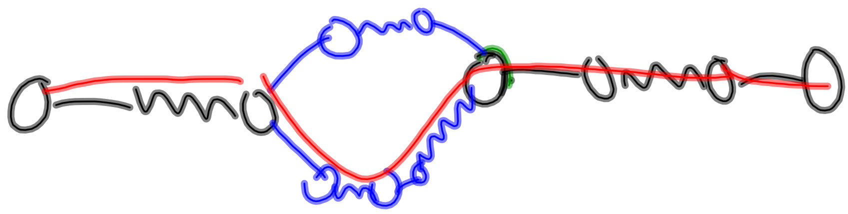
Augpath in G^c ⇒ Augpath in G.



contracted blossom

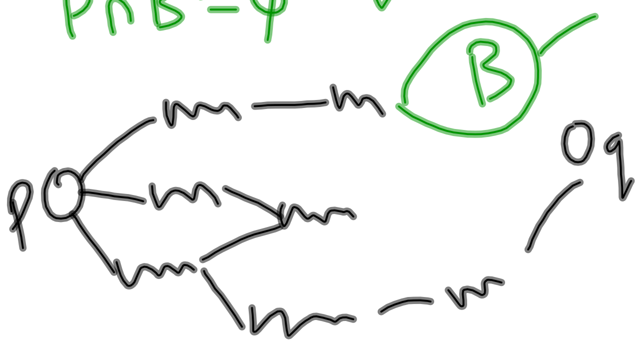


Apr 24-11:02 AM

Augpath in G ⇒ Augpath G^c

- Augpath P in G, from p to q, assume only p & q unmatched verts in G.
- blossom B

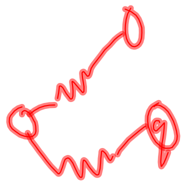
1) $P \cap B = \emptyset$ ✓



Apr 24-11:10 AM

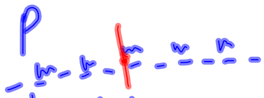
2) $P \cap B \neq \emptyset$

a) p is on the blossom



Let x be the last node in P that's also in B

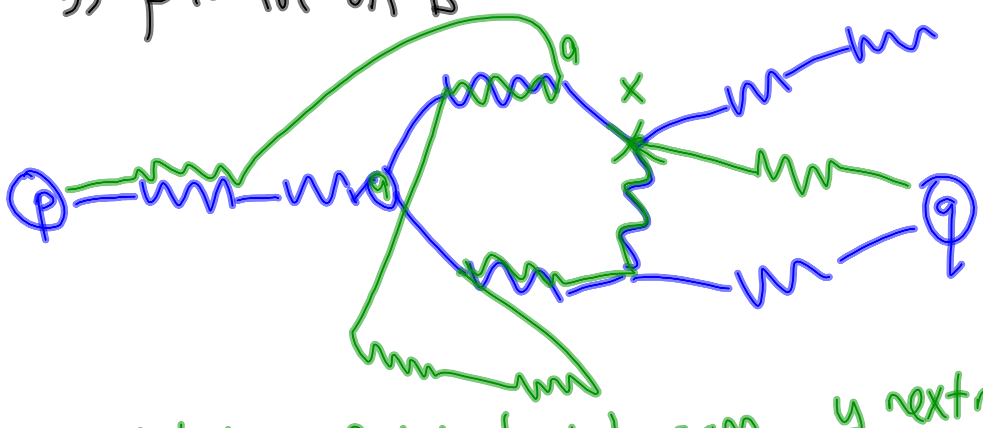
then $x \to q$, that is a suffix of P is an aug path in G^C



The edge out of x must be unmatched because
if $x \neq p$, there is another incident matched edge on B
if $x = p$, then because it is the first vertex on P, the first edge must be unmatched

Apr 24-11:15 AM

b) p is not on B



x is the last time P hit the blossom. y next node on P (x,y) must be unmatched.

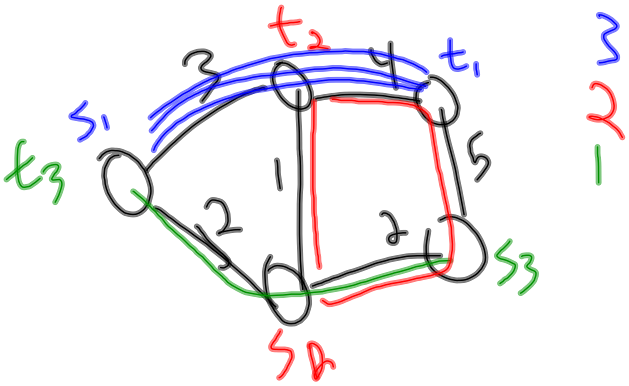
from aug. path $p \rightsquigarrow a, x \rightsquigarrow q$

Apr 24-11:25 AM

multi.pdf - Adobe Reader

Multicommodity Flow

- Given a directed network with edge capacities u and possibly costs c.
- Give a set K of k commodities, where a commodity i is defined by a triple (s_i, t_i, d_i) – source, sink and demand.
- For each commodity, you want to find a feasible flow, subject to joint capacity constraints.



Apr 24-11:43 AM

multi.pdf - Adobe Reader

Formulation

$f_i(v,w)$ is the flow of commodity i on edge (v,w).

$$\sum_w f_i(v,w) - \sum_w f_i(w,v) = \begin{cases} 0 & \text{if } v \neq s_i \text{ and } v \neq t_i \\ d_i & \text{if } v = s_i \\ -d_i & \text{if } v = t_i \end{cases} \quad \forall v \in V, i \in K$$

$$\sum_{i \in K} f_i(v,w) \leq u(v,w) \quad \forall (v,w) \in E$$

$$f_i(v,w) \geq 0 \quad \forall (v,w) \in E$$

- Single commodity flow: m variables, $m+n$ constraints
- Multicommoddity flow: km variables, $kn+m$ constraints, km non-negativity constraints

1000 nodes
3000 edges
100 comm.

Size of A matrix: $km(kn+m) = k^2nm + km^2$
A computationally challenging problem

3×10^6 var.
10^6 constr.

Apr 24-11:48 AM

multi.pdf - Adobe Reader

Facts About Multicommodity Flow

vars.

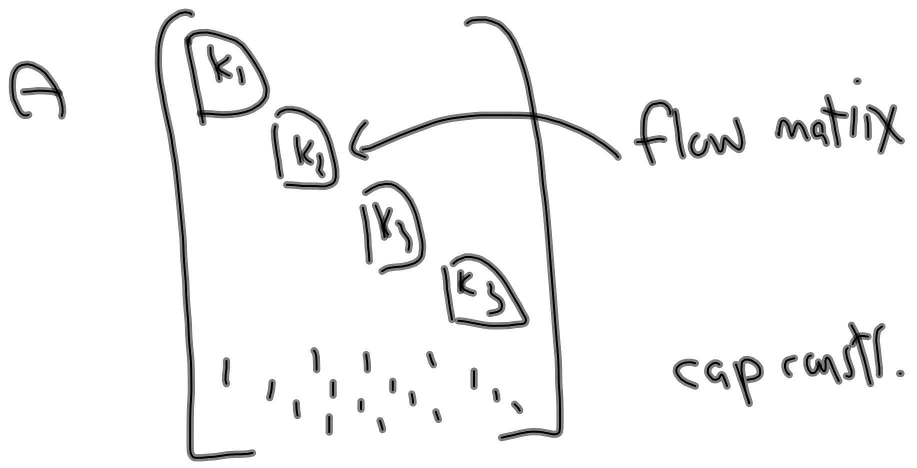
cons. [ ]

- LP is big
- A matrix is not Totally Unimodular.
- Optimal solution to a multicommodity flow LP might be fractional.
- All feasible solutions might be fractional.

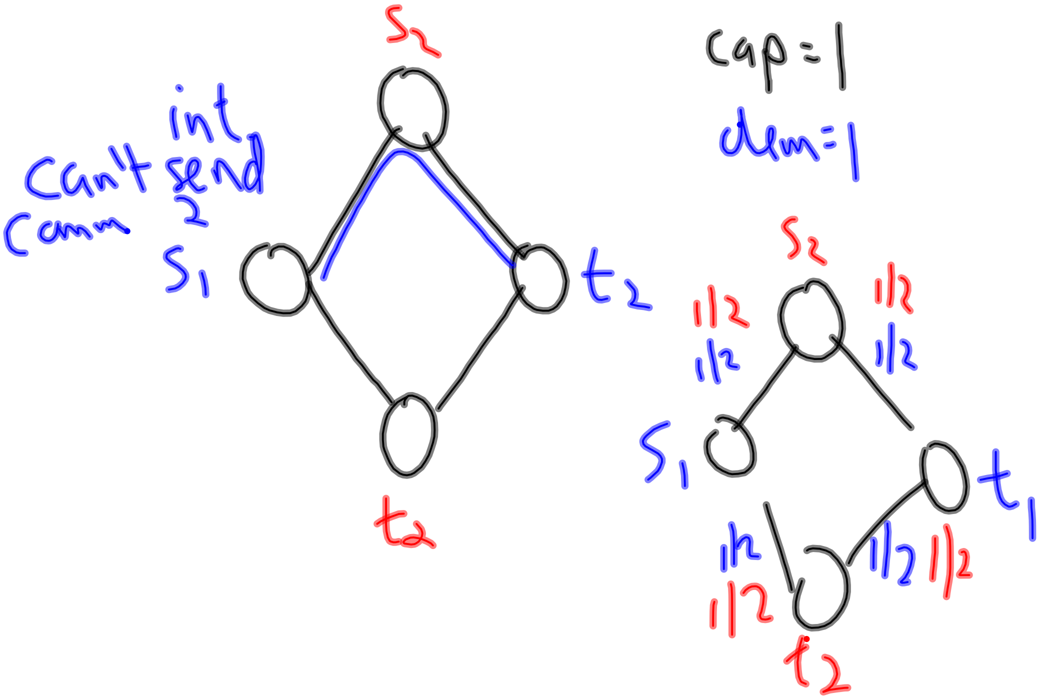
Optimization Variants

- Given costs on edges, $c(v,w)$, find a feasible flow minimizing $\Sigma_i \Sigma_{vw} c(v,w) f_i(v,w)$
- No given demands, maximize total flow
- No given demands maximize total flow cost
- Send at least z percent of each demand, maximize z . (concurrent flow)
- Send demands, find minimum α such that the flow is still feasible with capacities $\alpha c(v,w)$. (equivalent to previous problem)

Apr 24-11:49 AM

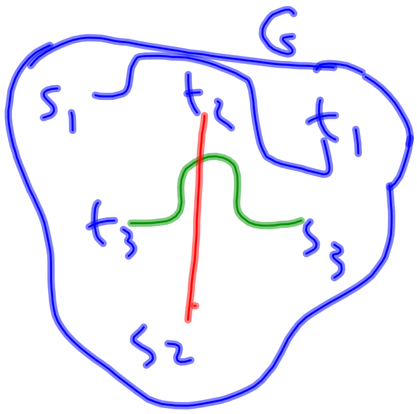


Apr 24-11:49 AM



Apr 24-11:51 AM

disjoint paths



Apr 24-12:05 PM