Recurrence Relations

Alg → Rec.

Rec → Solution

Binary search $T(n) = T(\frac{n}{2}) + 1$

Merge Sort $T(n) = 2T(\frac{n}{2}) + O(n)$

Boruvka $T(n,m) \le T(\frac{n}{a}, m-n) + O(m)$

$$T(n) \le \begin{cases} 2T(\lceil \frac{n}{2} \rceil) + O(n) & \text{if } n \ge 2 \\ O(1) & \text{o.w.} \end{cases}$$

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Solving Rec.

$T(n) = 2T(\frac{n}{2}) + O(n)$

$O(n \lg n)$

1. Guess & Check (pf. by induction)
2. Master Theorem $T(n) = aT(\frac{n}{b}) + f(n)$
3. Iterate / draw a tree

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$T(n) = 2T(\frac{n}{2}) + n$ rec.

Claim $T(n) \le cn \lg n$.

Pf by induction (base case trivial)

Assume for $n' < n$ that $T(n') \le cn' \lg n'$

$$\begin{aligned} T(n) &= 2T(\tfrac{n}{2}) + n \\ &\le 2c(\tfrac{n}{2})\lg(\tfrac{n}{2}) + n \\ &= cn(\lg n - 1) + n \\ &= cn \lg n - cn + n \\ &\le cn \lg n \quad \text{if } c \ge 1. \end{aligned}$$

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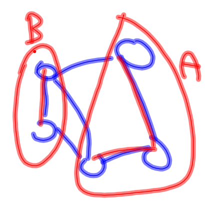

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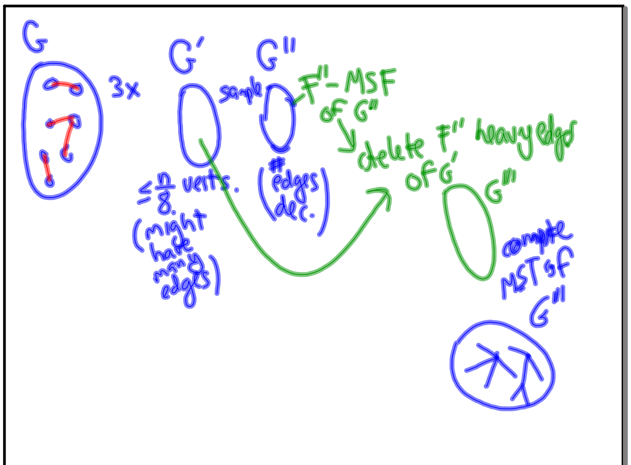

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Pf Think about Kruskal's

$e_1\ e_2\ e_3\ e_4\ e_5\ e_6\ e_7\ e_8 \dots$

(H) (H) (T) (H) (T) (T) (H) (T) (T) ...

our alg. not in st.

suppose e_3 would have been in MST. Then e_3 is going to be F-light

F-light edges = put in tree + Kruskal would have put in tree but flip T

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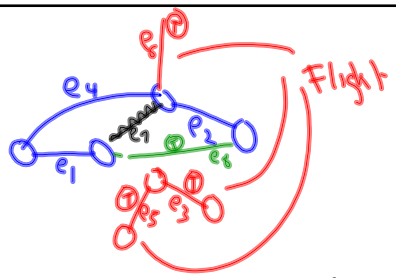

k k .. k

process will terminate (m exp) after $2(n-1)$ edges that Kruskal wants to all appear.

$\therefore \leq 2n$ F-light edges.

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Linear Program for MST

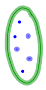
$$X_e = \begin{cases} 1 & \text{if } e \in MST \\ 0 & \text{o.w.} \end{cases}$$

$$\min \sum_{e \in E} c_e X_e$$

s.t. $\sum_{e \in E} X_e = n-1$

$\sum_{e \in S} X_e \leq |S|-1 \quad S \subseteq V$

$X_e \geq 0 \quad (X_e \in \{0,1\})$



$e \in S$

$e=(v,w)$

$v \in S$ & $w \in S$

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