Claim: If $p(v) = \overset{d(v)}{d(v_1, v)}$ then

$$\bar{c}(v,w) := c(v,w) + p(v) - p(w) \geq 0$$

Pf: Because $d()$ are s.p. distances

$$d(w) \leq d(v) + c(v,w) \quad \forall (v,w)$$

$$\Rightarrow c(v,w) + d(v) - d(w) \geq 0$$

$$\Rightarrow c(v,w) + p(v) - p(w) = \bar{c}(v,w) \geq 0$$

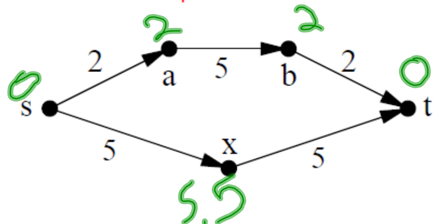
Feb 14-11:00 AM

http://www.columbia.edu/~cs2035/courses/ieor6614.S12/goldberg.pdf

File Edit Go to Favorites Help

Bidirectional Algorithm: Pitfalls

The algorithm is not as simple as it looks.



The searches meat at x, but x is not on the shortest path.

SOFSEM 07 8

Feb 14-11:16 AM

mmc.pdf - Adobe Reader

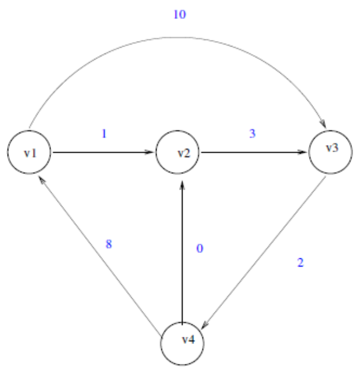
File Edit View Window Help

1 / 4 100% Tools Comment

$$\mu(X) = \frac{\Sigma_{vw \in X}\, c(v,w)}{|X|}$$

The minimum mean cycle is the cycle with smallest mean value.

$$\mu^* = \min_{\text{cycles } X} \mu(X)$$



2013

5/3

14/4

11.00 x 8.50 in

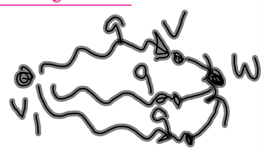
Feb 14-11:45 AM

mmc.pdf - Adobe Reader

File Edit View Window Help

2 / 4 100% Tools Comment

Computing a minimum mean cycle



Notes

- Computing the smallest value cycle is NP-hard
- The minimum mean cycle "approximates" the smallest value cycle.
- Choose v_1 as a "source"

Definition Let $d^k(v)$ be the length of a shortest directed walk from v_1 v containing exactly k edges. (∞ if no such walk exists).

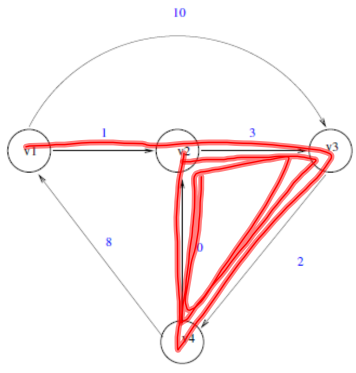
We can compute d^k for all k and via the recurrence:

$$d^k(w) = \min_{(v,w)\in E} d^{k-1}(v) + c(v,w)$$

- We initialize d^0 to ∞ for all vertices other than v_1 .

11.00 x 8.50 in

Feb 14-11:52 AM



$d^{10}(v_2) = 16$

	d^0	d^1	d^2	d^3	d^4	$\max(d^n(v) - d^k(v))/(n-k)$
v_1	0	∞	∞	20	14	7/2
v_2	∞	1	∞	12	6	5/3
v_3	∞	10	4	∞	15	11/2
v_4	∞	∞	12	6	∞	∞

Feb 14-11:55 AM

Pf Let $\mu^* =$ val of m.m.c.

Assume $\mu^* = 0$ (case 1)

$\Rightarrow$ no neg. cycles

$\Rightarrow$ At least 1 0-cost cycle

Compute shortest paths from v_1, call these dist's $d(v)$.

Def. $c^d(v,w) = c(v,w) + d(v) - d(w)$.

Feb 14-12:06 PM

Think about graph w/ edges costs c^d

$$c^d(v,w) \geq 0 \quad \forall (v,w)$$

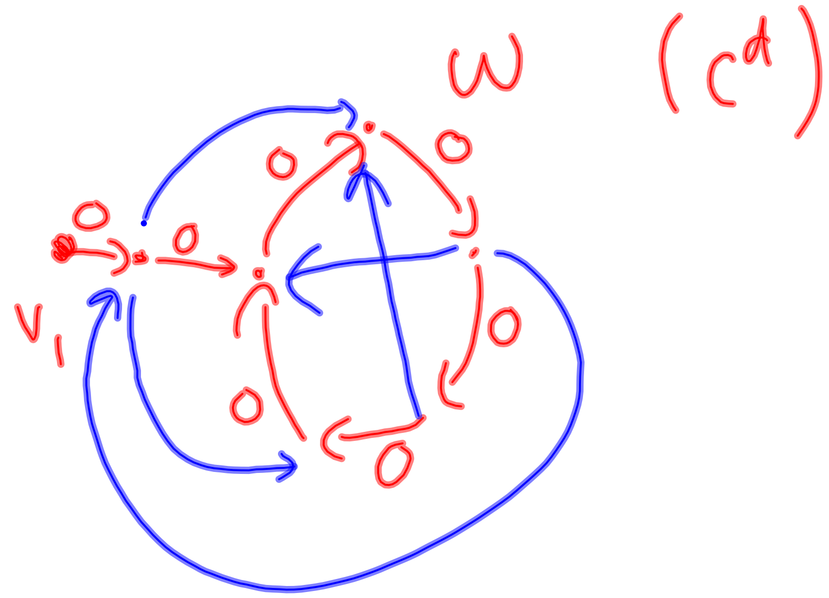
- let W be a 0-cost cycle w/r/t c.

$$c^d(v,w) = 0 \quad \forall (v,w) \in W$$

- If (v,w) is on a s.p. from v_1 to some vertex, then

$$c^d(v,w) = 0.$$

Feb 14-12:12 PM



Feb 14-12:16 PM