Carpool Fairness

	Days				
Person	**1**	**2**	**3**	**4**	**5**
1	X	X	X		
2	X		X		
3	X	X	X	X	X
4		X	X	X	X

What is a fair division of driving?

Carpool Fairness

Person	Days 1	2	3	4	5
1	X	X	X		
2	X		X		
3	X	X	X	X	X
4		X	X	X	X

What is a fair division of driving?

Person	Days 1	2	3	4	5	Total responsibility r_i
1	1/3	1/3	1/4			11/12
2	1/3		1/4			7/12
3	1/3	1/3	1/4	1/2	1/2	23/12
4		1/3	1/4	1/2	1/2	19/12

Proposal: Person i should drive no more than $\lceil r_i \rceil$ times.

Formaulation as a flow problem

	Days					
Person	**1**	**2**	**3**	**4**	**5**	**Total responsibility r_i**
1	**1/3**	**1/3**	**1/4**			**11/12**
2	**1/3**		**1/4**			**7/12**
3	**1/3**	**1/3**	**1/4**	**1/2**	**1/2**	**23/12**
4		**1/3**	**1/4**	**1/2**	**1/2**	**19/12**

- **Bipartite graph, nodes for each person and day.**
- **Think of r_i as supply for each person**
- **Think of 1 as demand for each day**
- **Edges between person and day if they can drive on that day..**

Does a flow of value 5 exist?

Fractional/Integral flow

- **A fractional flow of value 5 exists in graph with source-incident capacities $\lceil r_i \rceil$ and flow of r_i**
- **Theorem If capacities are integral a fractional flow of value x exists, then an integral flow for value $\lceil x \rceil$ exists.**
- **Use the integral flow to solve the carpool problem.**

Baseball Elimination

(SportsWriters end of Season Problem)

Team	Wins w_i	Games left g_i	Games against g_{ij}			
			NY	Bos	Tor	Bal
NY Yankees	93	8	-	1	6	1
Boston Red Sox	89	4	1	-	0	3
Toronto Blue Jays	88	7	6	0	-	1
Baltimore Orioles	86	5	1	3	1	-

Question: Which teams are eliminated and which are not?

Formalism

- w_i - wins for team i
- g_i - games left for team i
- g_{ij} - games left between i and j

For any subset R of teams T :

- wins in R, $w(R) = \Sigma_{i \in R} w_i$
- games left in R, $g(R) = \Sigma_{i,j \in R, i<j} g_{ij}$

A lower bound on a number of teams that some team must win

$$a(R) = \frac{w(R) + g(R)}{|R|}$$

Claim: For $i \in T$, $R \subseteq T - \{i\}$, and $a(R) > w_i + g_i$, then i is eliminated.

Justification: Some team must win the average.

A stronger condition?

Let x_{ij} **be the number of times that** i **beats** j **.**

Team k **is not eliminated if there exist** x_{ij} **s.t. it is possible for team** k **to come in first. That is,**

$$x_{ij} + x_j = g_{ij} \quad \forall i, j \in T \tag{1}$$

$$w_k + g_k \geq w_i + \Sigma_{j \in T - \{k\}} x_{ij} \quad \forall i \in T \tag{2}$$

$$x_{ij} \geq 0 \quad x_{ij} \in \{0, 1\} \tag{3}$$

Can also use flow problem.

Proof that flow problem solves the problem

Claims:

- **If a flow** f **of value** $\sum_{i<j} g_{ij}$ **exists, then team** k **is not eliminated.**
- **If no flow of value** $\sum_{i<j} g_{ij}$ **exists then them** k **is eliminated.**