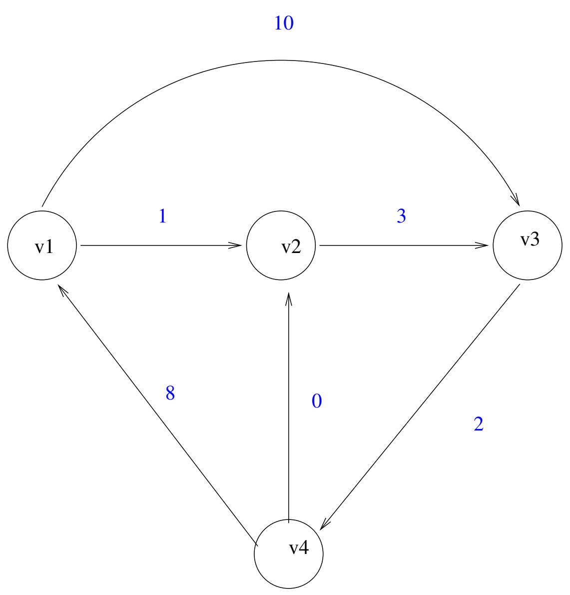
Minimum Mean Cycle

- **Given a strongly connected graph** $G = (V, E)$ **, with edge weights** c **.**
- **Let** $|X|$ **be the number of edges (vertices) on the cycle**
- **For any cycle X, the mean value of X is defined by**

$$\mu(X) = \frac{\Sigma_{vw \in X}\, c(v, w)}{|X|}$$

The minimum mean cycle is the cycle with smallest mean value.

$$\mu^* = \min_{\textbf{cycles } X} \mu(X)$$



Computing a minimum mean cycle

Notes

- Computing the smallest value cycle is NP-hard
- The minimum mean cycle "approximates" the smallest value cycle.
- Choose v_1 as a "source"

Definition Let $d^k(v)$ be the length of a shortest directed walk from v_1 to v containing exactly k edges. (∞ if no such walk exists).

We can compute d^k for all k and via the recurrence:

$$d^k(w) = \min_{(v,w)\in E} d^{k-1}(v) + c(v,w)$$

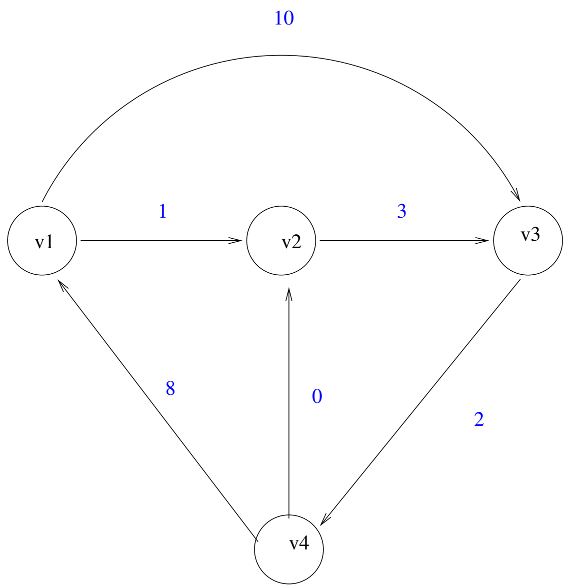
- We initialize d^0 to ∞ for all vertices other than v_1 .
- We can compute $d^k(v)$ for all vertices v and all $k \leq n$ in $O(nm)$ time, by iterating the recurrence.

Computing the minimum mean cycle

$$\mu^* = \min_{v \in V} \max_{0 \le k \le n-1} \left\{ \frac{d^n(v) - d^k(v)}{n - k} \right\}$$

Example

$$\mu^* = \min_{v \in V} \max_{0 \le k \le n-1} \left\{ \frac{d^n(v) - d^k(v)}{n-k} \right\}$$



	d^0	d^1	d^2	d^3	d^4	$\max(d^n(v) - d^k(v))/(n-k)$
v_1	0	∞	∞	20	14	7/2
v_2	∞	1	∞	12	6	5/3
v_3	∞	10	4	∞	15	11/2
v_4	∞	∞	12	6	∞	∞