Flow Decomposition

Def: **A feasible flow obeys capacity, non-negativity, and flow conservation constraints.**

$$\min \sum_{(v,w)\in E} c(v,w)f(v,w)$$

subject to

$$\begin{aligned} f(v,w) &\leq u(v,w) && \forall (v,w) \in E \\ \sum_{w\in V} f(v,w) - \sum_{w\in V} f(w,v) &= b(v) && \forall v \in V \\ f(v,w) &\geq 0 && \forall (v,w) \in E \end{aligned}$$

Flow decomposition **Any flow f can be decomposed into paths and cycles such that**

- **Every directed path with positive flow connects a deficit node (** $b(v) < 0$ **) to a surplus node (** $b(v) > 0$ **) .**
- **At most** $n+m$ **paths and cycles have non-zero flow; out of these at most** m **cyclces have non-zero flow.**

Augmenting Cycle Theorem

- **Let** f **and** g **be any two feasible flows. Then** f **can be obtained from** g **by augmenting along at most** m **directed cycles in** G_g **.**
- **The cost of** f **is the cost of** g **plus the sum of the costs of the cycles.**