Robust Optimal Power Flow with Uncertain Renewables

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Dimacs Workshop on Energy Infrastructure
THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid’s Limits

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.
Large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid).

Large power deviations in renewables must be balanced by other sources, which may be far away.

Flow reversals may be observed – control difficult.

A solution – expand transmission capacity! Difficult (expensive), takes a long time.

Problems already observed when renewable penetration high.
CIGRE - International Conference on Large High Voltage Electric Systems '09

- "Fluctuations" – 15-minute timespan
- Due to turbulence ("storm cut-off")
- Variation of the same order of magnitude as mean
- Most problematic when renewable penetration starts to exceed 20 – 30%
- Many countries are getting into this regime
Optimal power flow (economic dispatch, tertiary control)

- Used periodically to handle the next time window (e.g. 15 minutes, one hour)
- Choose generator outputs
- Minimize cost (quadratic)
- Satisfy demands, meet generator and network constraints
- Constant load (demand) estimates for the time window
OPF:

\[
\min \ c(p) \quad \text{ (a quadratic)}
\]

s.t.

\[
B\theta = p - d
\]  \hspace{1cm} (1)

\[
|y_{ij}(\theta_i - \theta_j)| \leq u_{ij} \quad \text{for each line } ij
\]  \hspace{1cm} (2)

\[
P_{g}^{\text{min}} \leq p_g \leq P_{g}^{\text{max}} \quad \text{for each bus } g
\]  \hspace{1cm} (3)

Notation:

\(p\) = vector of generations \(\in \mathcal{R}^n\), \(d\) = vector of loads \(\in \mathcal{R}^n\)

\(B\in \mathcal{R}^{n\times n}\), \(\text{(bus susceptance matrix)}\)

\(\forall i, j:\quad B_{ij} = \begin{cases} 
-y_{ij}, & ij \in \mathcal{E} \ (\text{set of lines}) \\
\sum_{k;\{k,j\}\in\mathcal{E}}y_{kj}, & i = j \\
0, & \text{otherwise}
\end{cases}\)
min \ c(p) \ (a \ quadratic) \\
\ s.t. \\
\ B\theta = p - d \\
\ |y_{ij}(\theta_i - \theta_j)| \leq u_{ij} \ \text{for each line } ij \\
\ P_{g}^{\min} \leq p_g \leq P_{g}^{\max} \ \text{for each bus } g
\[
\begin{align*}
\min & \quad c(p) \quad \text{(a quadratic)} \\
\text{s.t.} & \quad B\theta = p - d \\
& \quad |y_{ij}(\theta_i - \theta_j)| \leq u_{ij} \quad \text{for each line } ij \\
& \quad P_g^{\min} \leq p_g \leq P_g^{\max} \quad \text{for each bus } g
\end{align*}
\]

How does OPF handle short-term fluctuations in demand (d)?

**Frequency control:**

- Automatic control: primary, secondary
- Generator output varies up or down *proportionally* to aggregate change

How does OPF handle short-term fluctuations in renewable output?

**Answer:** Same mechanism, now used to handle aggregate wind power change
Wind model?

Need to model variation in wind power between dispatches.

Wind at farm attached to bus $i$ of the form $\mu_i + w_i$ – Weibull distribution?
Wind model

From CIGRE report, aggregated over Germany:
Experiment

Bonneville Power Administration data, Northwest US
- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time
Line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may ‘trip’. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction contributes to line temperature
- In medium-length lines (∼ 100 miles) the line limit is due to voltage drop, not thermal reasons
- In long lines, it is due to phase angle change (stability), not thermal reasons
- In 2003 U.S. blackout event, many critical lines tripped due to thermal reasons, but well short of their line limit
summary: exceeding limit for too long is bad, but complicated
want: "fraction time a line exceeds its limit is small"
proxy: \( \text{prob(violation on line } i) < \epsilon \) for each line \( i \)
Goals

- simple control
- aware of limits
- not too conservative
- computationally practicable
Control

For each generator $i$, two parameters:

- $p_i$ = mean output
- $\alpha_i$ = response parameter

Real-time output of generator $i$:

$$p_i = \bar{p}_i - \alpha_i \sum_j \Delta \omega_j$$

where $\Delta \omega_j$ = change in output of renewable $j$ (from mean).

$$\sum_i \alpha_i = 1$$

$\sim$ primary + secondary control
Set up control

average case

low wind

high wind

Bienstock, Harnett, Chertkov
Robust Optimal Power Flow with Uncertain Renewables
Computing line flows

wind power at bus $i$: $\mu_i + w_i$

DC approximation

- $B\theta = \bar{p} - d$
- $+ (\mu + w - \alpha \sum_{i \in G} w_i)$
- $\theta = B^+(\bar{p} - d + \mu) + B^+(I - \alpha e^T)w$

flow is a linear combination of bus power injections:

$$f_{ij} = y_{ij}(\theta_i - \theta_j)$$
Computing line flows

\[ f_{ij} = y_{ij} \left( (B_i^+ - B_j^+)^T (\bar{p} - d + \mu) + (A_i - A_j)^T w \right) , \]

\[ A = B^+(I - \alpha e^T) \]

Given distribution of wind can calculate moments of line flows:

- \( Ef_{ij} = y_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu) \)
- \( \text{var}(f_{ij}) := s_{ij}^2 \geq y_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2 \) (assuming independence)
- and higher moments if necessary
Chance constraints to deterministic constraints

- chance constraint: \( P(f_{ij} > f_{ij}^{max}) < \epsilon_{ij} \) and \( P(f_{ij} < -f_{ij}^{max}) < \epsilon_{ij} \)

- from moments of \( f_{ij} \), can get conservative approximations using e.g. Chebyshev’s inequality
Chance constraints to deterministic constraints

- chance constraint: $P(f_{ij} > f_{ij}^{\text{max}}) < \epsilon_{ij}$ and $P(f_{ij} < -f_{ij}^{\text{max}}) < \epsilon_{ij}$

- from moments of $f_{ij}$, can get conservative approximations using e.g. Chebyshev’s inequality

- for Gaussian wind, can do better, since $f_{ij}$ is Gaussian:

  $$|Ef_{ij}| + \text{var}(f_{ij})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}}$$
Formulation:
Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

\[
\min_{\bar{\mu}, \alpha} \mathbb{E}[c(\bar{\mu})]
\]

s.t. \[
\sum_{i \in G} \alpha_i = 1, \quad \alpha \geq 0
\]

\[
B\delta = \alpha, \quad \delta_n = 0
\]

\[
\sum_{i \in G} \bar{\mu}_i + \sum_{i \in W} \mu_i = \sum_{i \in D} d_i
\]

\[
\bar{f}_{ij} = y_{ij}(\bar{\theta}_i - \bar{\theta}_j),
\]

\[
B\bar{\theta} = \bar{\mu} + \mu - d, \quad \bar{\theta}_n = 0
\]

\[
s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2
\]

\[
|\bar{f}_{ij}| + s_{ij} \phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\max}
\]
Data errors?

\[ s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2 \]

\[ |\bar{f}_{ij}| + s_{ij} \phi^{-1} (1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}} \]

(the $\bar{f}_{ij}$ implicitly incorporate the $\mu_i$)
Data errors?

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(the \( \bar{f}_{ij} \) implicitly incorporate the \( \mu_i \))

What if the \( \mu_i \) or the \( \sigma_k \) are incorrect? ... What happens to

\[ \text{Prob}(f_{ij} > u_{ij})? \]
Let the *correct* parameters be \( \tilde{\mu}_i, \tilde{\sigma}_i \) for each farm \( i \).
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**Theorem:** Suppose there are parameters $M > 0$, $V > 0$ such that

$$|\bar{\mu}_i - \mu_i| < M \mu_i \text{ and } |\bar{\sigma}_i^2 - \sigma_i| < V \sigma_i$$

for all $i$. Then:

$$\text{Prob}(f_{ij} > f_{\text{max}}) < \epsilon_{ij} + O(V) + O(M)$$

Here, the $O()$ "hides" some constants dependent on e.g. reactances.

Can we guarantee that $\text{Prob}(f_{ij} > f_{\text{max}})$ is small even under data errors?
Let the correct parameters be $\tilde{\mu}_i, \tilde{\sigma}_i$ for each farm $i$.

**Theorem:** Suppose there are parameters $M > 0$, $V > 0$ such that

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$$\text{Prob}(f_{ij} > f_{ij}^{\text{max}}) < \epsilon_{ij} + O(V) + O(M)$$
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Can we guarantee that $\text{Prob}(f_{ij} > f_{ij}^{\text{max}})$ is small even under data errors?
Polyhedral data error model:
\[ |\tilde{\sigma}_i - \sigma_i| \leq \gamma_i \ \forall i, \ \sum_i \frac{|\tilde{\sigma}_i - \sigma_i|}{\gamma_i} \leq \Gamma. \]

Ellipsoidal data error model:
\[ (\tilde{\sigma} - \sigma)^T A (\tilde{\sigma} - \sigma) \leq b \]

Here $A \succeq 0$ and $b > 0$ are parameters.
Nominal case:

\[
E \left| f_{ij} \right| + \text{var}(f_{ij}) \phi - 1 (1 - \epsilon_{ij}) \leq f_{\text{max}}_{ij} \rightarrow \text{a conic constraint}
\]
chance constraints

Nominal case:  \[ |E f_{ij}| + \text{var}(f_{ij}) \phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}} \]
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\[ \rightarrow \text{a conic constraint} \]
chance constraints

Nominal case: $|E f_{ij}| + \text{var}(f_{ij})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\max}$

$\rightarrow$ a conic constraint

Robust case: $\max_{\mathcal{E}} \{|E f_{ij}| + \text{var}(f_{ij})\phi^{-1}(1 - \epsilon_{ij})\} \leq f_{ij}^{\max}$

($\mathcal{E}$ : data error model)
chance constraints

Nominal case: \[ |E f_{ij}| + \operatorname{var}(f_{ij}) \phi^{-1} (1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}} \]

→ a conic constraint

Robust case: \[ \max_{\mathcal{E}} \{ |E f_{ij}| + \operatorname{var}(f_{ij}) \phi^{-1} (1 - \epsilon_{ij}) \} \leq f_{ij}^{\text{max}} \]

( \mathcal{E} : data error model) how to solve?
chance constraints

Nominal case: $|E f_{ij}| + var(f_{ij})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}}$

$\rightarrow$ a conic constraint

Robust case: $\max_{\mathcal{E}} \{ |E f_{ij}| + var(f_{ij})\phi^{-1}(1 - \epsilon_{ij}) \} \leq f_{ij}^{\text{max}}$

( $\mathcal{E}$ : data error model) how to solve?

**Theorem.** The robust problem is a convex optimization problem and can be solved in polynomial time in the polyhedral and ellipsoidal data cases.
Nominal case: $|E f_{ij}| + \text{var}(f_{ij})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\max}$

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($\mathcal{E}$ : data error model) how to solve?

**Theorem.** The robust problem is a convex optimization problem and can be solved in polynomial time in the polyhedral and ellipsoidal data cases.

An “ambiguous chance-constrained problem”
Toy example

1. What if no line limits?
2. What if tight limit on line connecting generators?
Answer 1

What if no line limits?

total demand: 100
cost: 5720
Answer 2

What if small limit on line connecting generators?

total demand: 100
cost: 5774.8
Experiment

How much wind penetration can we handle?
And how much money does this save?

39-bus New England system from MATPOWER
Experiment

'standard' OPF solution with 10% buffer on line limits feasible only up to 5% penetration (right)

Cost 1,275,000 – almost 5 times greater than chance-constrained
Polish system - winter 2003-04 evening peak
Big cases

Polish 2003-2004 winter peak case
- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source

CPLEX: the optimization problem has
- 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 nonzeros, 87 dense columns
Big cases

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CPLEX: the optimization problem has
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Big cases

**CPLEX:**
- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

**Gurobi:**
- time: 31.1 seconds
- "Numerical trouble encountered"
Cutting-plane method

overview

Cutting-plane algorithm:

remove all conic constraints
repeat until convergence:
    solve linearly constrained problem
    if no conic constraints violated: return
    find separating hyperplane for maximum violation
    add linear constraint to problem
Cutting-plane method

Candidate solution violates conic constraint
Cutting-plane method

Separate: find a linear constraint also violated
Solve again with linear constraint
Cutting-plane method

New solution still violates conic constraint
Cutting-plane method

Separate again
Cutting-plane method

We might end up with many linear constraints
Cutting-plane method

... which approximate the conic constraint
conic constraint:

\[
\sqrt{x_1^2 + x_2^2 + \ldots + x_k^2} = \|x\|_2 \leq y
\]

candidate solution:

\[(x^*, y^*)\]

cutting-plane (linear constraint):

\[
\|x^*\|_2 + \frac{x^* T}{\|x^*\|_2} (x - x^*) = \frac{x^* T x}{\|x^*\|_2} \leq y
\]
Polish 2003-2004 case
CPLEX: “opt status 6”
Gurobi: “numerical trouble”

Example run of cutting-plane algorithm:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Max rel. error</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2e-1</td>
<td>7.0933e6</td>
</tr>
<tr>
<td>4</td>
<td>1.3e-3</td>
<td>7.0934e6</td>
</tr>
<tr>
<td>7</td>
<td>1.9e-3</td>
<td>7.0934e6</td>
</tr>
<tr>
<td>10</td>
<td>1.0e-4</td>
<td>7.0964e6</td>
</tr>
<tr>
<td>12</td>
<td>8.9e-7</td>
<td>7.0965e6</td>
</tr>
</tbody>
</table>

Total running time: 32.9 seconds
Back to motivating example

BPA case

- standard OPF: cost 235603, 7 lines unsafe \( \geq 8\% \) of the time
- CC-OPF: cost 237297, every line safe \( \geq 98\% \) of the time
- run time = 9.5 seconds (one cutting plane!)
Back to motivating example

BPA case

- standard OPF: cost 235603, 7 lines unsafe \( \geq 8\% \) of the time
- CC-OPF: cost 237297, every line safe \( \geq 98\% \) of the time
- run time = 9.5 seconds (one cutting plane!)
Our chance-constrained optimal power flow:

- safely accounts for variability in wind power between dispatches
- uses a simple control which is easily integrable into existing system
- is fast enough to be useful at the appropriate time scale