Multi-time-step Chance Constrained Generation Re-dispatch

Scott Backhaus, Russell Bent, Daniel Bienstock, Misha Chertkov

Columbia University, LANL

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Organization

1. Review: chance-constrained OPF
2. Extension: Robustness
3. Extension: Multi-time-step formulation

Review of past work: chance-constrained DC OPF

- CIGRE '09: large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid)
- Large power deviations in renewables must be balanced by other sources, which may be far away
- Flow reversals may be observed – control difficult
- A solution – expand transmission capacity! Difficult (expensive), takes a long time
- Problems **already observed** when renewable penetration high

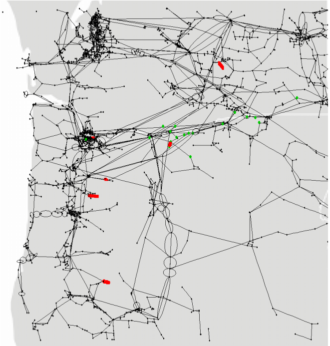
CIGRE -International Conference on Large High Voltage Electric Systems '09

- "Fluctuations" – 15-minute timespan
- Due to turbulence ("storm cut-off")
- Variation of the same order of magnitude as mean
- Most problematic when renewable penetration starts to exceed $20-30\%$
- Many countries are getting into this regime

Experiment

Bonneville Power Administration data, Northwest US

- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



DC-OPF:

$$\min \; c(p) \quad \text{(a quadratic)}$$

s.t.

$$B\theta = p - d \tag{1}$$

$$|\beta_{ij}(\theta_i - \theta_j)| \;\leq\; u_{ij} \quad \text{for each line } ij \tag{2}$$

$$P_g^{min} \;\leq\; p_g \;\leq\; P_g^{max} \quad \text{for each generator } g \tag{3}$$

Notation:

p = vector of generations $\in \mathbb{R}^n$, d = vector of loads $\in \mathbb{R}^n$

$B \in \mathbb{R}^{n \times n}$, (bus susceptance matrix)

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- $\overline{p_i}$ = mean output
- α_i = response parameter ("participation factor")

Real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

where

$$\sum_i \alpha_i = 1, \quad \alpha \geq 0$$

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$\sim$ primary + secondary control, extends existing practice

Modeling risk: line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction contributes to line temperature
- IEEE Standard 738 computes line temperature as a function of power flow and **numerous** exogenous parameters (wind, temperature, humidity, air pressure, date, time of day, latitude and longitude, ...)
- In 2003 U.S. blackout event, many critical lines tripped due to thermal reasons, but well short of their line limit

Modeling risk: line limits and line tripping

summary: exceeding limit for too long is bad, but precise model difficult

want: "fraction time a line exceeds its limit is small"

proxy: prob(violation on line pq) $< \epsilon_{pq}$

Computing line flows

wind power at bus i: $\mu_i + \mathbf{w}_i$

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d$
  $+(\mu + \mathbf{w} - \alpha \sum_{i \in G} \mathbf{w}_i)$
- $\boldsymbol{\theta} = B^{+}(\bar{p} - d + \mu) + B^{+}(I - \alpha e^{T})\mathbf{w}$
- flow is a linear combination of bus power injections:

$$\mathbf{f_{ij}} = \beta_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

Computing line flows

$$\mathbf{f_{ij}} = \beta_{ij}\left((B_i^+ - B_j^+)^T(\bar{p} - d + \mu) + (A_i - A_j)^T\mathbf{w}\right),$$

$$A = B^+(I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

- $E\mathbf{f_{ij}} = \beta_{ij}(B_i^+ - B_j^+)^T(\bar{p} - d + \mu)$
- $var(\mathbf{f_{ij}}) := s_{ij}^2 \geq \beta_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2$
  (assuming independence)
- and higher moments if necessary

Chance constraints to deterministic constraints

- chance constraint: $P(\mathbf{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\mathbf{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality

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- from moments of $\mathbf{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\mathbf{f_{ij}}$ is Gaussian :

$$|E\mathbf{f_{ij}}| + var(\mathbf{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

Formulation:

Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1, \ \alpha \geq 0$$

$$B\delta = \alpha, \delta_n = 0$$

$$\sum_{i \in G} \overline{p}_i + \sum_{i \in \mathcal{F}} \mu_{\mathbf{i}} = \sum_{i \in D} d_i$$

$$\overline{f}_{ij} = \beta_{ij}(\overline{\theta}_i - \overline{\theta}_j),$$

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A convex optimization problem.

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- Solvers (Cplex, Gurobi) **cannot** solve problem
- Specialized cutting-plane algorithm solves in $\sim$ 30 seconds on normal computer

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Soon to appear in *SIAM Review*

Need for robustness!

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3. When data errors are **small** we want our solutions to degrade **very little** from nominal behavior

Sensitivity to data errors?

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What if the $\boldsymbol{\mu_i}$ or the $\boldsymbol{\sigma_k}$ are incorrect? ... What happens to

$$Prob(\mathbf{f_{ij}} > f_{ij}^{max})?$$

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Theorem: Suppose there are parameters $M > 0$, $V > 0$ such that

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In other words, solution quality degrades "gracefully"

Robustness: small errors

Polyhedral data error model:

$$|\tilde{\sigma}_i^2 - \sigma_i^2| \leq \gamma_i \ \forall i, \quad \sum_i \frac{|\tilde{\sigma}_i^2 - \sigma_i^2|}{\gamma_i} \leq \Gamma.$$

Ellipsoidal data error model:

$$(\tilde{\sigma}^2 - \sigma^2)^T A (\tilde{\sigma}^2 - \sigma^2) \ \leq \ b$$

Here $A \succeq 0$ and $b > 0$ are parameters.

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How NOT to formulate: duality

Lemma: Let

$$U(\gamma, \Gamma) \quad = \quad \left\{ \sigma^2 \in \mathbb{R}_+^{\mathcal{F}} \, : \, |\sigma_i^2 - \bar{\sigma}_i^2| \, \leq \, \gamma_i \; \forall i \in \mathcal{F}, \;\; \sum_{i \in \mathcal{F}} \frac{|\sigma_i^2 - \bar{\sigma}_i^2|}{\gamma_i} \leq \Gamma \right\}.$$

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NOT CONVEX!

But the original constraint IS convex!

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Algorithm.

1. Solve convex relaxation (initially: empty). Let δ^* be optimal.

2. (For each line (i,j)) compute

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which is a convex problem in the above cases. Let $\{\hat{\sigma}^2\}$ be optimal.

3. Linearize (5) around δ^* and $\{\hat{\sigma}^2\}$ (and add cut)

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2. We will set the average operating point $\bar{p}_i^{(h)}$ for each generator i and (end of) interval h. $\longrightarrow$ This allows us to model generator ramping.
3. Each interval h split into $K \geq 1$ *sub-intervals*.
4. Actual expected output of generator i at subinterval k of interval h:

$$\bar{p}_i^{(h,k)} \quad = \quad \frac{K-k}{K-1}\bar{p}_i^{(h)} + \frac{k-1}{K-1}\bar{p}_i^{(h+1)}$$

5. $\boldsymbol{\alpha_i^{(h,k)}} \ \boldsymbol{=}$ participation factor for generator i at subinterval k, interval h. Could be $\boldsymbol{\alpha_i^{(h)}}$. Or even $\boldsymbol{\alpha_i}$.
6. $\mathbf{d_i^{(h)}} \ =$ estimate for demand at bus i at interval h.

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We can take advantage of the possibility of the recourse in the formulation