Operations Research Problems in Power Engineering

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Power engineering for non-power engineers

References

Andersson: *Modelling and Analysis of Electric Power Systems*

Bergen, Vittal: *Power Systems Analysis*

Glover, Sarma, Overbye: *Power System Analysis and Design*
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References

Andersson: *Modelling and Analysis of Electric Power Systems*

Bergen, Vittal: *Power Systems Analysis*

Glover, Sarma, Overbye: *Power System Analysis and Design*

Rebours, Kirschen: *What is spinning reserve?*
Power engineering for non-power engineers
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- Conductor
- Steam magnetic field
- Stator
- Rotor
- Source
- Energy
- Current, voltage

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A generator produces **current** at a certain **voltage**.
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**Ohm’s law:** power = current × voltage
AC Power Flows

Voltage at bus \( k \):
\[ v_k(t) = V_{max} k \cos(\omega t + \theta_V) \]

Current injected at \( k \) into \( m \):
\[ i_{km}(t) = I_{max} km \cos(\omega t + \theta_I) \]

Power injected at \( k \) into \( m \):
\[ p_{km}(t) = v_k(t) i_{km}(t) \]

Averaged over period \( T \):
\[ p_{km} = \frac{1}{T} \int_0^T p_{km}(t) \, dt = \frac{1}{2} V_{max} k I_{max} km \cos(\theta_V - \theta_I) \]
AC Power Flows

Real-time:

Voltage at bus $k$:

$$v_k(t) = V_{\text{max}} \cos(\omega t + \theta_V).$$

Current injected at $k$ into $km$:

$$i_{km}(t) = I_{\text{max}} \cos(\omega t + \theta_{I_{km}}).$$

Power injected at $k$ into $km$:

$$p_{km}(t) = v_k(t)i_{km}(t).$$

Averaged over period $T$:

$$p_{km} = \frac{1}{T} \int_0^T p_{km}(t) \, dt = \frac{1}{2} V_{\text{max}} I_{\text{max}} \cos(\theta_V - \theta_{I_{km}}).$$
AC Power Flows

Real-time:

- Voltage at bus $k$: $v_k(t) = V_{k}^{\text{max}} \cos(\omega t + \theta_V)$
- Current injected at $k$ into $km$: $i_{km}(t) = I_{km}^{\text{max}} \cos(\omega t + \theta_{I_{km}})$. 
Real-time:

- Voltage at bus $k$: $v_k(t) = V_{k}^{\text{max}} \cos(\omega t + \theta_V^k)$
- Current injected at $k$ into $km$: $i_{km}(t) = I_{km}^{\text{max}} \cos(\omega t + \theta_{km}^l)$.
- Power injected at $k$ into $km$: $p_{km}(t) = v_k(t)i_{km}(t)$. 
AC Power Flows

Real-time:

- Voltage at bus $k$: $v_k(t) = V_k^{\text{max}} \cos(\omega t + \theta_k^V)$
- Current injected at $k$ into $km$: $i_{km}(t) = I_{km}^{\text{max}} \cos(\omega t + \theta_{km}^I)$
- Power injected at $k$ into $km$: $p_{km}(t) = v_k(t)i_{km}(t)$

Averaged over period $T$:

- $p_{km} = \frac{1}{T} \int_0^T p(t)$
AC Power Flows

Real-time:

- Voltage at bus $k$: $v_k(t) = V_k^{max} \cos(\omega t + \theta_k^V)$
- Current injected at $k$ into $km$: $i_{km}(t) = I_{km}^{max} \cos(\omega t + \theta_{km}^I)$.
- Power injected at $k$ into $km$: $p_{km}(t) = v_k(t)i_{km}(t)$.

Averaged over period $T$:

- $p_{km} = \frac{1}{T} \int_0^T p(t) = \frac{1}{2} V_k^{max} I_{km}^{max} \cos(\theta_k^V - \theta_{km}^I)$. 
\[ p_{km} = \frac{1}{T} \int_0^T p(t) = \frac{1}{2} V^\text{max}_k I^\text{max}_{km} \cos(\theta^V_k - \theta^I_{km}) \]
\[ p_{km} = \frac{1}{T} \int_0^T p(t) = \frac{1}{2} V_k^{\text{max}} I_{km}^{\text{max}} \cos(\theta_k - \theta_{km}) \]

\[ v_k(t) = V_k^{\text{max}} \Re e^{i(\omega t + \theta_k)} , \quad i_{km}(t) = I_{km}^{\text{max}} \Re e^{i(\omega t + \theta_{km})} \]
\[ p_{km} = \frac{1}{T} \int_0^T p(t) = \frac{1}{2} V_{km}^{max} l_{km}^{max} \cos(\theta_V - \theta_I) \]

\[ v_k(t) = V_k^{max} \Re e^{i(\omega t + \theta_V)} \]

\[ i_{km}(t) = l_{km}^{max} \Re e^{i(\omega t + \theta_I)} \]

\[ V_k = \frac{V_k^{max}}{\sqrt{2}} e^{j\theta_v} \]

\[ I_{km} = \frac{l_{km}^{max}}{\sqrt{2}} e^{j\theta_{I}} \]
\[ p_{km} = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{2} V_{km}^\text{max} I_{km}^\text{max} \cos(\theta_k^V - \theta_k^I) \]

\[ v_k(t) = V_k^\text{max} \Re e^{j(\omega t + \theta_k^V)}, \quad i_{km}(t) = I_{km}^\text{max} \Re e^{j(\omega t + \theta_{km}^I)} \]

\[ V_k = \frac{V_{km}^\text{max}}{\sqrt{2}} e^{j\theta_k^V}, \quad I_{km} = \frac{I_{km}^\text{max}}{\sqrt{2}} e^{j\theta_{mk}^I} \]

\[ p_{km} = |V_k||I_{km}| \cos(\theta_k^V - \theta_k^I) = \Re(V_k I_{km}^*) \]
\[ p_{km} = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{2} V_{k}^{\text{max}} I_{km}^{\text{max}} \cos(\theta_k^V - \theta_l^{km}) \]

\[ v_k(t) = V_k^{\text{max}} \Re e^{i(\omega t + \theta_k^V)}, \quad i_{km}(t) = I_{km}^{\text{max}} \Re e^{i(\omega t + \theta_l^{km})} \]

\[ V_k = \frac{V_k^{\text{max}}}{\sqrt{2}} e^{i\theta_k^V}, \quad I_{km} = \frac{I_{km}^{\text{max}}}{\sqrt{2}} e^{i\theta_l^{km}} \]

\[ p_{km} = |V_k||I_{km}| \cos(\theta_k^V - \theta_l^{km}) = \Re (V_k I_{km}^*) \]

\[ q_{km} = \Im (V_k I_{km}^*) \quad \text{and} \quad S_{km} = p_{km} + jq_{km} \]
\[ V_k = \frac{V_k^{\text{max}}}{\sqrt{2}} e^{j\theta_k}, \quad I_{km} = \frac{I_{km}^{\text{max}}}{\sqrt{2}} e^{j\theta_{mk}} \quad \text{(voltage, current)} \]

\[ p_{km} = \Re(V_k I_{km}^*) , \quad q_{km} = \Im(V_{km} I_{km}^*) \]

(1)
\[ V_k = \frac{V_{k}^{\text{max}}}{\sqrt{2}} e^{j\theta_k} \], \quad l_{km} = \frac{l_{km}^{\text{max}}}{\sqrt{2}} e^{j\theta_{lmk}} \text{ (voltage, current)}

\[ p_{km} = \mathcal{R}e(V_k l_{km}^*), \quad q_{km} = \mathcal{I}m(V_{km} l_{km}^*) \]  \hspace{1cm} (1)

\[ l_{km} = y_{\{k,m\}}(V_k - V_m), \]
\[ V_k = \frac{V_k^{\text{max}}}{\sqrt{2}} e^{j\theta_k}, \quad I_{km} = \frac{I_{km}^{\text{max}}}{\sqrt{2}} e^{j\theta_{mk}} \] (voltage, current)

\[ p_{km} = \Re(V_k I_{km}^*), \quad q_{km} = \Im(V_{km} I_{km}^*) \] (1)

\[ I_{km} = y_{\{k,m\}}(V_k - V_m), \quad y_{\{k,m\}} = \text{admittance of } km. \] (2)
\[ V_k = \frac{V_{k}^{\text{max}}}{\sqrt{2}} e^{j\theta_k} \]
\[ I_{km} = \frac{I_{km}^{\text{max}}}{\sqrt{2}} e^{j\theta_{mk}} \] (voltage, current)

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\[ q_{km} = \Im(V_{km} I_{km}^*) \] (1)

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\[ y\{k,m\} = \text{admittance of } km. \] (2)

Network Equations
\begin{align*}
V_k & = \frac{V_{k}^{\text{max}}}{\sqrt{2}} e^{j\theta_{k}} , \quad I_{km} = \frac{I_{km}^{\text{max}}}{\sqrt{2}} e^{j\theta_{mk}} \text{ (voltage, current)} \\
p_{km} & = \Re(V_k I_{km}^*), \quad q_{km} = \Im(V_k I_{km}^*) \quad (3) \\
l_{km} & = y_{\{k,m\}}(V_k - V_m), \quad y_{\{k,m\}} = \text{admittance of } km. \quad (4)
\end{align*}

\textbf{Network Equations}

\begin{align*}
\sum_{km \in \delta(k)} p_{km} &= \hat{P}_k , \quad \sum_{km \in \delta(k)} q_{km} = \hat{Q}_k \quad \forall k \quad (5)
\end{align*}
\[ V_k = \frac{V_{\text{max}}}{\sqrt{2}} e^{j\theta_k}, \quad I_{km} = \frac{I_{\text{max}}^{\text{km}}}{\sqrt{2}} e^{j\theta^{\text{km}}} \] (voltage, current)

\[ p_{km} = \Re(V_k I_{km}^*), \quad q_{km} = \Im(V_{km} I_{km}^*) \] (3)

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Network Equations

\[ \sum_{km \in \delta(k)} p_{km} = \hat{P}_k, \quad \sum_{km \in \delta(k)} q_{km} = \hat{Q}_k \quad \forall \ k \] (5)

**Generator:** \( \hat{P}_k, |V_k| \) given. Other buses: \( \hat{P}_k, \hat{Q}_k \) given.
Managing changing demands

Average Hourly Load, PJM Mid-Atlantic Region

UK Electricity Demand and Wind Generation April 2015

8 days lull 347 GWh
267 GWh recharge
9 days lull 339 GWh

Energy Matters
euanmears.com
Data: BM reports / Gridwatch
QLD1 5 minute Demand and Price for period 19/02/2016 00:00 to 20/02/2016 02:35

- RRP (Regional Reference Price)
- Total Demand

03:00-04:00: RRP and Total Demand are at their lowest levels.
04:00-05:00: RRP and Total Demand start to increase.
05:00-06:00: RRP and Total Demand continue to rise, with some fluctuations.
06:00-07:00: RRP and Total Demand reach their peak.
07:00-08:00: RRP and Total Demand start to decrease, with RRP showing more significant drops.
08:00-09:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
09:00-10:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
10:00-11:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
11:00-12:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
12:00-13:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
13:00-14:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
14:00-15:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
15:00-16:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
16:00-17:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
17:00-18:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
18:00-19:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
19:00-20:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
20:00-21:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
21:00-22:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
22:00-23:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
23:00-00:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
00:00-01:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
01:00-02:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.
02:00-03:00: RRP and Total Demand continue to decrease, with RRP showing more significant drops.

Overall, the graph shows a clear pattern of demand and price changes throughout the day, with peaks and drops at specific times.
What happens when there is a generation/load mismatch
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Frequency response:

\[ \Delta P \Rightarrow \text{frequency change } \Delta \omega \approx -c \Delta P \]
What happens when there is a generation/load mismatch

Frequency response:
mismatch \( \Delta P \)
What happens when there is a generation/load mismatch

Frequency response:
mismatch \( \Delta P \) \( \Rightarrow \) frequency change \( \Delta \omega \approx -c \Delta P \)
Managing changing demands

1. Primary frequency control. Handles instantaneous (small) changes.

2. Secondary control. Handles changes that span more than a few seconds. Agent: algorithms, pre-set controls.


4. Once (?) a day: unit commitment problem. Chooses which generators will operate in the next day or half-day. Agent: algorithms, humans.
Managing changing demands

1. Primary frequency control. Handles instantaneous (small) changes. **Agent:** physics.

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3. "Tertiary" control: OPF (Optimal power flow). Manages longer lasting changes. Run every few minutes. **Goal:** economic generation that meets demands while maintaining feasibility. **Agent:** algorithmic computations, humans.

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THE ENERGY CHALLENGE

Wind Energy Bumps Into Power Grid’s Limits

The Maple Ridge Wind farm near Lowville, N.Y. It has been forced to shut down when regional electric lines become congested.

By MATTHEW L. WALD
Published: August 26, 2008

When the builders of the Maple Ridge Wind farm spent $320 million to put nearly 200 wind turbines in upstate New York, the idea was to get paid for producing electricity. But at times, regional electric lines have been so congested that Maple Ridge has been forced to shut down even with a brisk wind blowing.
Large unexpected fluctuations in wind power can cause additional flows through the transmission system (grid).

Large power deviations in renewables must be balanced by other sources, which may be far away.

Flow reversals may be observed – control difficult.

A solution – expand transmission capacity! Difficult (expensive), takes a long time.

Problems already observed when renewable penetration high.
“Fluctuations” – 15-minute timespan

Due to turbulence ("storm cut-off")

Variation of the same order of magnitude as mean

Most problematic when renewable penetration starts to exceed 20 – 30%

Many countries are getting into this regime
Optimal power flow (economic dispatch, tertiary control)

- Used periodically to handle the next time window (e.g. 15 minutes, one hour)
- Choose generator outputs
- Minimize cost (quadratic)
- Satisfy demands, meet generator and network constraints
- Constant load (demand) estimates for the time window
**OPF:**

\[
\min \ c(p) \quad \text{(a quadratic)}
\]

s.t.

\[B\theta = p - d\] \hspace{1cm} (6)

\[|y_{ij}(\theta_i - \theta_j)| \leq u_{ij} \quad \text{for each line } ij\] \hspace{1cm} (7)

\[P_{g_{\text{min}}} \leq p_g \leq P_{g_{\text{max}}} \quad \text{for each bus } g\] \hspace{1cm} (8)

**Notation:**

\[p = \text{vector of generations } \in \mathbb{R}^n, \quad d = \text{vector of loads } \in \mathbb{R}^n\]

\[B \in \mathbb{R}^{n \times n}, \quad \text{(bus susceptance matrix)}\]

\[\forall i, j: \quad B_{ij} = \begin{cases} 
-y_{ij}, \quad &ij \in \mathcal{E} \ (\text{set of lines}) \\
\sum_{k; \{k,j\} \in \mathcal{E}} y_{kj}, \quad &i = j \\
0, \quad &\text{otherwise}
\end{cases}\]
\[
\begin{align*}
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\text{s.t.} \quad B\theta &= p - d \\
|y_{ij}(\theta_i - \theta_j)| & \leq u_{ij} \quad \text{for each line } ij \\
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\end{align*}
\]
\[ \text{min } c(p) \quad (\text{a quadratic}) \]
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\[ B\theta = p - d \]
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How does the grid handle short-term fluctuations in demand \(d\)?

**Secondary frequency control:**

- Deployed a few seconds after ongoing change – “minute-by-minute” control

- Generator output varies up or down proportionally to aggregate change

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\end{align*} \]

How does the grid handle short-term fluctuations in demand (d)?

**Secondary frequency control:**

- Deployed a few seconds after ongoing change – “minute-by-minute” control
- Generator output varies up or down proportionally to aggregate change

How does the grid handle short-term fluctuations in renewable output? **Answer:** Same mechanism, now used to handle aggregate wind power change
“Participation factors”

For each generator $i$, a parameter $\alpha_i$ with
- $\sum_i \alpha_i = 1$
- $\alpha_i \geq 0$
- $\alpha_i > 0$ only for selected generators

Assuming real-time generation/demand mismatch $\Delta$, real-time output of generator $i$:

$$p_i = \bar{p}_i - \alpha_i \Delta$$

where $\bar{p}_i$ = OPF computed output for generator $i$.

**Note:** the $\alpha_i$ are precomputed e.g. uniform or based on economic considerations.
Experiment

Bonneville Power Administration data, Northwest US
- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time
Wind model?

Need to model variation in wind power between dispatches

Wind at farm attached to bus $i$ of the form $\mu_i + w_i$. Weibull distribution?
Typical wind farm spans a significant geographical zone with many turbines
Wind model

- Typical wind farm spans a significant geographical zone with many turbines
- Real-time variations due to turbulence
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- Real-time variations due to turbulence.
- Turbulence is local ($\approx 50m$ radius) and arguably local effects are independent.
Wind model

- Typical wind farm spans a significant geographical zone with many turbines
- Real-time variations due to turbulence
- Turbulence is local ($\approx 50m$ radius) and arguably local effects are independent
- Working model: real-time variations in a farm’s output modeled as a normal variable
Line limits and line tripping

If power flow in a line exceeds its limit, the line becomes compromised and may 'trip'. But process is complex and time-averaged:

- Thermal limit is most common
- Thermal limit may be in terms of terminal equipment, not line itself
- Wind strength and wind direction contributes to line temperature
- IEEE Standard 738 attempts to account for everything
- In 2003 U.S. blackout event, many critical lines tripped due to thermal reasons, but well short of their line limit
Background

- When a power line overheats it becomes exposed to several risk factors
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- If the line overheats enough, it may sag and experience a contact/arc, which will cause a trip
- If overheating is detected, and is deemed risky, the line will may be preemptively tripped
Background

- When a power line overheats it becomes exposed to several risk factors
- If the line overheats enough, it may sag and experience a contact/arc, which will cause a trip
- If overheating is detected, and is deemed risky, the line will may be preemptively tripped
- What is risky?
When a power line overheats it becomes exposed to several risk factors.

- If the line overheats enough, it may sag and experience a contact/arc, which will cause a trip.
- If overheating is detected, and is deemed risky, the line will may be preemptively tripped.
- What is risky? What is a critical temperature?
When a power line overheats it becomes exposed to several risk factors.

If the line overheats enough, it may sag and experience a contact/arc, which will cause a trip.

If overheating is detected, and is deemed risky, the line will may be preemptively tripped.

What is risky? What is a critical temperature?

2003 event: critical temperatures estimates were sometimes incorrect.
IEEE Standard 738

A comprehensive method for determining the temperature of a power line,
IEEE Standard 738

- A comprehensive method for determining the temperature of a power line, as a function of current and pause physical properties of the conductor.

Note: power lines can be more than 100 miles long.

How can we account for data uncertainty, errors, unavailability?
IEEE Standard 738

- A comprehensive method for determining the temperature of a power line, as a function of current and pause physical properties of the conductor.
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- It also relies on the heat equation for a “static” calculation.
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- It also relies on the heat equation for a “static” calculation.
- Note: power lines can be more than 100 miles long.
- How can we account for data uncertainty, errors, unavailability?
Line trip model

summary: exceeding limit for too long is bad, but complicated
want: "fraction time a line exceeds its limit is small"
proxy: \( \text{prob}(\text{violation on line } i) < \epsilon \) for each line \( i \)
Goals

- simple control
- aware of limits
- not too conservative
- computationally practicable with a simple algorithm
For each generator $i$, two parameters:

- $\bar{p}_i = $ risk-aware mean output
- $\alpha_i = $ risk-aware participation factor

Real-time output of generator $i$:

$$p_i = \bar{p}_i - \alpha_i \sum_{j} \Delta \omega_j$$

where $\Delta \omega_j = $ change in output of renewable $j$ (from mean).

$$\sum_{i} \alpha_i = 1$$
Set up control

- **average case**
  - +20
  - -120
  - 0
  - +50
  - α = 0.8
  - +50

- **low wind**
  - +0
  - -120
  - 0
  - +50
  - α = 0.8
  - +66

- **high wind**
  - +40
  - -120
  - 0
  - +54
  - α = 0.8
  - +34
Computing line flows

wind power at bus $i$: $\mu_i + w_i$

DC approximation

$B\theta = \bar{p} - d$
$+ (\mu + w - \alpha \sum_{i \in G} w_i)$

$\theta = B^+ (\bar{p} - d + \mu) + B^+ (I - \alpha e^T)w$

flow is a linear combination of bus power injections:

$$f_{ij} = y_{ij} (\theta_i - \theta_j)$$
Computing line flows

\[ f_{ij} = y_{ij} \left( (B_i^+ - B_j^+)^T (\bar{p} - d + \mu) + (A_i - A_j)^T w \right), \]

\[ A = B^+(I - \alpha e^T) \]

Given distribution of wind can calculate moments of line flows:

- \( E f_{ij} = y_{ij} (B_i^+ - B_j^+)^T (\bar{p} - d + \mu) \)

- \( \text{var}(f_{ij}) := s_{ij}^2 \geq y_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2 \) (assuming independence)

- and higher moments if necessary
Chance constraints to deterministic constraints

- chance constraint: $P(f_{ij} > f_{ij}^{\text{max}}) < \epsilon_{ij}$ and $P(f_{ij} < -f_{ij}^{\text{max}}) < \epsilon_{ij}$

- from moments of $f_{ij}$, can get conservative approximations using e.g. Chebyshev’s inequality
Chance constraints to deterministic constraints

- chance constraint: \( P(f_{ij} > f_{ij}^{\text{max}}) < \epsilon_{ij} \) and \( P(f_{ij} < -f_{ij}^{\text{max}}) < \epsilon_{ij} \)

- from moments of \( f_{ij} \), can get conservative approximations using e.g. Chebyshev’s inequality

- for Gaussian wind, can do better, since \( f_{ij} \) is Gaussian:

\[
|E f_{ij}| + \text{var}(f_{ij}) \phi^{-1} (1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}}
\]
Formulation:
Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

\[
\min_{\bar{p},\alpha} \mathbb{E}[c(\bar{p})]
\]

s.t.
\[
\sum_{i \in G} \alpha_i = 1, \quad \alpha \geq 0
\]
\[
B\delta = \alpha, \quad \delta_n = 0
\]
\[
\sum_{i \in G} \bar{p}_i + \sum_{i \in W} \mu_i = \sum_{i \in D} d_i
\]
\[
\bar{f}_{ij} = y_{ij}(\bar{\theta}_i - \bar{\theta}_j),
\]
\[
B\bar{\theta} = \bar{p} + \mu - d, \quad \bar{\theta}_n = 0
\]
\[
s_{ij}^2 \geq y_{ij}^2 \sum_{k \in W} \sigma_k^2 (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2
\]
\[
|\bar{f}_{ij}| + s_{ij} \phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{\text{max}}
\]
Big cases

Polish 2003-2004 winter peak case
■ 2746 buses, 3514 branches, 8 wind sources
■ 5% penetration and $\sigma = .3\mu$ each source

CPLEX: the optimization problem has
■ 36625 variables
■ 38507 constraints, 6242 conic constraints
■ 128538 nonzeros, 87 dense columns
Big cases

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- 5% penetration and $\sigma = 0.3\mu$ each source

CPLEX: the optimization problem has
- 36625 variables
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Big cases

CPLEX:
- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

Gurobi:
- time: 31.1 seconds
- "Numerical trouble encountered"
Big cases

CPLEX:
- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

Gurobi:
- time: 31.1 seconds
- "Numerical trouble encountered"

→ basic cutting-plane algorithm works well
Experiment: Polish grid, 20% wind penetration, 50 farms

B., Chertkov, Harnett, SIAM Review '15
Extensions and Ongoing work

- Account for errors in estimations of distribution
Extensions and Ongoing work

- Account for errors in estimations of distribution
- Account for correlations (spatial and timewise)
Extensions and Ongoing work

- Account for errors in estimations of distribution
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- Extension to unit commitment problem
Extensions and Ongoing work

- Account for errors in estimations of distribution
- Account for correlations (spatial and timewise)
- Extension to unit commitment problem
- Better risk model for line temperature
  B., Blanchet and Li ’15
The heat equation on a 1-dimensional line

- Line modeled as one-dimensional object parameterized by $x$, $0 \leq x \leq L$.
- Time domain: $[0, \tau]$
The heat equation on a 1-dimensional line

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The heat equation on a 1-dimensional line

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- Time domain: $[0, \tau]$ (for example: OPF intervals)
- $I(t) = \text{current at time } t$, $T(x, t) = \text{temperature at } x \text{ at time } t$. 

Heat equation: 

$$
\frac{\partial T(x, t)}{\partial t} = \kappa \frac{\partial^2 T(x, t)}{\partial x^2} + \alpha I^2(t) - \nu (T(x, t) - T_{ext}(x, t)),
$$

where $\kappa \geq 0$, $\alpha \geq 0$ and $\nu \geq 0$ are (line dependent) constants, and $T_{ext}(x, t)$ is the ambient temperature at $(x, t)$. 
The heat equation on a 1-dimensional line

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The heat equation on a 1-dimensional line

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The heat equation on a 1-dimensional line

Heat equation:

\[
\frac{\partial T(x, t)}{\partial t} = \kappa \frac{\partial^2 T(x, t)}{\partial x^2} + \alpha l^2(t) - \nu(T(x, t) - T_{\text{ext}}(x, t)).
\]

IEEE 738, other authors:

\[
\frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - T_{\text{ext}}(x, t)).
\]

Us:

\[
\frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))).
\]

\(h(x)\) is a random variable, at \(x\).
The heat equation on a 1-dimensional line

Heat equation:

$$\frac{\partial T(x, t)}{\partial t} = \kappa \frac{\partial^2 T(x, t)}{\partial x^2} + \alpha l^2(t) - \nu(T(x, t) - T^{\text{ext}}(x, t)).$$

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$$h(x) = \text{a random variable, at } x.$$
Back to the stochastic heat equation

\[ \frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))). \]

Recall: \( x \in [0, L], \ t \in [0, \tau] \)
Back to the stochastic heat equation

\[ \frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))). \]

Recall: \( x \in [0, L], \ t \in [0, \tau] \)

Integrate and divide by \( L \), get

\[ \frac{1}{L} \int_{0}^{L} \frac{\partial T(x, t)}{\partial t} \ dx = \alpha l^2(t) - \frac{\nu}{L} \int_{0}^{L} T(x, t) \ dx + \frac{\nu}{L} \int_{0}^{L} G(h(x)) \ dx. \]
Back to the stochastic heat equation

\[
\frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))).
\]

Recall: \( x \in [0, L], t \in [0, \tau] \)

Integrate and divide by \( L \), get

\[
\frac{1}{L} \int_0^L \frac{\partial T(x, t)}{\partial t} \, dx = \alpha l^2(t) - \frac{\nu}{L} \int_0^L T(x, t) \, dx + \frac{\nu}{L} \int_0^L G(h(x)) \, dx.
\]
Back to the stochastic heat equation

\[ \frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))). \]

Recall: \( x \in [0, L], \ t \in [0, \tau] \)

Integrate and divide by \( L \), get

\[ \frac{1}{L} \int_0^L \frac{\partial T(x, t)}{\partial t} \, dx = \alpha l^2(t) - \frac{\nu}{L} \int_0^L T(x, t) \, dx + \frac{\nu}{L} \int_0^L G(h(x)) \, dx. \]

\[ \frac{1}{L} \int_0^L \frac{\partial T(x, t)}{\partial t} \, dx = \frac{d}{dt} \frac{1}{L} \int_0^L T(x, t) \, dx = \frac{dH(t)}{dt}. \]

\( H(t) \triangleq \frac{1}{L} \int_0^L T(x, t) \, dx \) (average internal line temperature at \( t \))
Back to the stochastic heat equation

\[
\frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu (T(x, t) - G(h(x))).
\]

Recall: \(x \in [0, L], \ t \in [0, \tau]\)

Integrate and divide by \(L\), get

\[
\frac{dH(t)}{dt} = \alpha l^2(t) - \nu H(t) + \frac{\nu}{L} \int_0^L G(h(x)) \, dx.
\]

\(R \triangleq \frac{1}{L} \int_0^L G(h(x)) \, dx\) \hspace{1em} (average ambient temperature,
Back to the stochastic heat equation

\[
\frac{\partial T(x, t)}{\partial t} = \alpha I^2(t) - \nu (T(x, t) - G(h(x))).
\]

Recall: \( x \in [0, L], \ t \in [0, \tau] \)

Integrate and divide by \( L \), get

\[
\frac{dH(t)}{dt} = \alpha I^2(t) - \nu H(t) + \frac{\nu}{L} \int_0^L G(h(x)) \, dx.
\]

\[ R \triangleq \frac{1}{L} \int_0^L G(h(x)) \, dx \] (average ambient temperature, random!)

Bienstock Columbia University
Operations Research Problems in Power Engineering
Back to the stochastic heat equation

\[
\frac{\partial T(x, t)}{\partial t} = \alpha l^2(t) - \nu(T(x, t) - G(h(x))).
\]

Recall: \( x \in [0, L], \ t \in [0, \tau] \)

Integrate and divide by \( L \), get

\[
\frac{dH(t)}{dt} = \alpha l^2(t) - \nu H(t) + \frac{\nu}{L} \int_0^L G(h(x))dx.
\]

\[ R \triangleq \frac{1}{L} \int_0^L G(h(x))dx \quad \text{(average ambient temperature, random!)} \]

\[
\frac{dH(t)}{dt} = \alpha l^2(t) - \nu H(t) + \nu R.
\]
Once more

\[
\frac{dH(t)}{dt} = \alpha I^2(t) - \nu H(t) + \nu R.
\]

\[
H(t) \triangleq \frac{1}{L} \int_0^L T(x, t) \, dx, \quad R \triangleq \frac{1}{L} \int_0^L G(h(x)) \, dx,
\]
Once more

\[
\frac{dH(t)}{dt} = \alpha l^2(t) - \nu H(t) + \nu R.
\]

\[
H(t) \triangleq \frac{1}{L} \int_0^L T(x, t) dx, \quad R \triangleq \frac{1}{L} \int_0^L G(h(x)) dx,
\]

Solution:

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha l^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
\]

where

\[
C = H(0) = \frac{1}{L} \int_0^L T(x, 0) dx.
\]
Once more

\[
\frac{dH(t)}{dt} = \alpha l^2(t) - \nu H(t) + \nu R.
\]

\[
H(t) \triangleq \frac{1}{L} \int_0^L T(x, t) dx, \quad R \triangleq \frac{1}{L} \int_0^L G(h(x)) dx,
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\[
C = H(0) = \frac{1}{L} \int_0^L T(x, 0) dx.
\]

**Control goal:** make \( I(t) \) “large”,
Once more

\[
\frac{dH(t)}{dt} = \alpha I^2(t) - \nu H(t) + \nu R.
\]

\[
H(t) \triangleq \frac{1}{L} \int_0^L T(x, t) \, dx, \quad R \triangleq \frac{1}{L} \int_0^L G(h(x)) \, dx,
\]

**Solution:**

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha I^2(s) \, ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
\]

where

\[
C = H(0) = \frac{1}{L} \int_0^L T(x, 0) \, dx.
\]

**Control goal:** make \( I(t) \) “large”, but with \( P \left( \max_{t \in [0, \tau]} H(t) > k \right) \leq \epsilon \)
Constant control: \( l(t) = \bar{l}, \) for all \( t \in [0, \tau] \)
Constant control: \( I(t) = \bar{I}, \text{ for all } t \in [0, \tau] \)

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{I}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
\]

where

\[
C = H(0) = \frac{1}{L} \int_0^L T(x, 0) dx.
\]

Constant current \( \Rightarrow H(t) = \left( \frac{\alpha}{\nu} \bar{I}^2 + R \right)(1 - e^{-\nu t}) + Ce^{-\nu t} \)
Constant control: \( I(t) = \bar{I}, \text{ for all } t \in [0, \tau] \)

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{I}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
\]

where

\[
C = H(0) = \frac{1}{L} \int_0^L T(x, 0) dx.
\]

Constant current \( \Rightarrow H(t) = (\frac{\alpha}{\nu} \bar{I}^2 + R)(1 - e^{-\nu t}) + Ce^{-\nu t} \)

So, \( H'(t) > 0 \) for \( \bar{I} \) large enough,
Constant control: \( l(t) = \bar{l} \), for all \( t \in [0, \tau] \)

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{l}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
\]

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C = H(0) = \frac{1}{L} \int_0^L T(x, 0) dx.
\]

Constant current \( \Rightarrow H(t) = (\frac{\alpha}{\nu} \bar{l}^2 + R)(1 - e^{-\nu t}) + Ce^{-\nu t} \)

So, \( H'(t) > 0 \) for \( \bar{l} \) large enough, (and of constant sign for any \( \bar{l} \)).
Constant control: \( I(t) = \bar{I} \), for all \( t \in [0, \tau] \)

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H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{I}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
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So, \( H'(t) > 0 \) for \( \bar{I} \) large enough, (and of constant sign for any \( \bar{I} \)).

So, \( P\left(\max_{t \in [0, \tau]} H(t) > k\right) \leq \epsilon \) equivalent to \( P(H(\tau) > k) \leq \epsilon. \)
Constant control: \( I(t) = \bar{I}, \text{ for all } t \in [0, \tau] \)

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H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{I}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
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where

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Constant current \( \Rightarrow H(t) = (\frac{\alpha}{\nu} \bar{I}^2 + R)(1 - e^{-\nu t}) + Ce^{-\nu t} \)

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So, \( P \left( \max_{t \in [0, \tau]} H(t) > k \right) \leq \epsilon \) equivalent to \( P(H(\tau) > k) \leq \epsilon \).

Solution:

\[
\bar{I}^2 \leq \frac{\nu k - Ce^{-\nu \tau} - \rho \epsilon (1 - e^{-\nu \tau})}{\alpha \left(1 - e^{-\nu \tau}\right)}
\]
Constant control: \( I(t) = \bar{I} \), for all \( t \in [0, \tau] \)

\[
H(t) = \int_0^t e^{-\nu(t-s)} \alpha \bar{I}^2(s) ds + R(1 - e^{-\nu t}) + Ce^{-\nu t},
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So, \( P(\max_{t \in [0, \tau]} H(t) > k) \leq \epsilon \) equivalent to \( P(H(\tau) > k) \leq \epsilon. \)

Solution:

\[
\bar{I}^2 \leq \frac{\nu k - Ce^{-\nu \tau} - \rho \epsilon (1 - e^{-\nu \tau})}{\alpha \frac{1 - e^{-\nu \tau}}{1 - e^{-\nu \tau}}} = L(\tau, k)
\]
Adaptive control

1. At time $\tau = 0$, we compute values $I_1$ and $I_2$, for $i = 1, 2, \ldots, n$.

   These values are used as follows:

2. For all $t \in [0, \tau/2]$, we set $I(t) = I_1$.

3. At time $\tau/2$, we observe the value of $R$. Assuming $R = r_i$, then for all $t \in [\tau/2, \tau]$, we set $I(t) = I_2$.

Goals:

(a) $P(H(\tau) > k) < \epsilon$.

(b) $I_1 \leq L(\tau/2)$.

(c) What about performance?
Adaptive control

**Simplification:**

$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).
Adaptive control

Simplification:

\( R \) is a discrete random variable. \( P(R = r_i) = p_i, i = 1, 2, \ldots, n \) (known).

1. At time \( \tau = 0 \), we compute values \( l_1 \) and \( l_{2,i} \) for \( i = 1, 2, \ldots, n \).

These values are used as follows:
Adaptive control

Simplification:
$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

1. At time $\tau = 0$, we compute values $l_1$, and $l_{2,i}$ for $i = 1, 2, \ldots, n$. These values are used as follows:

2. For all $t \in [0, \tau/2]$, we set $l(t) = l_1$. 

Goals:
(a) $P(H(\tau) > k) < \epsilon$.
(b) $l_1 \leq L(\tau/2)$.
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**Simplification:**
$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

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**Goals:**

(a) $P(H(\tau) > k) < \epsilon$. 
Adaptive control

**Simplification:**

$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

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3. At time $\tau/2$, we observe the value of $R$. Assuming $R = r_i$, then for all $t \in [\tau/2, \tau]$, we set $I(t) = I_{2,i}$.

**Goals:**

(a) $P(H(\tau) > k) < \epsilon$. $k$ smaller than critical temperature

(b) $I_1 \leq L(\tau/2)$. 
Adaptive control

Simplification:

$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

1. At time $\tau = 0$, we compute values $l_1, i$, and $l_{2,i}$ for $i = 1, 2, \ldots, n$. These values are used as follows:
2. For all $t \in [0, \tau/2]$, we set $l(t) = l_1$.
3. At time $\tau/2$, we observe the value of $R$. Assuming $R = r_i$, then for all $t \in [\tau/2, \tau]$, we set $l(t) = l_{2,i}$.

Goals:

(a) $P(H(\tau) > k) < \epsilon$.  \textit{k smaller than} critical temperature
(b) $l_1 \leq L(\tau/2)$.
(c) What about performance?
We want to maximize:

- “Total” current: $\frac{T}{2} l_1 + \frac{T}{2} l_{2,i}$
We want to maximize:

- “Total” current: $\frac{T}{2} \ l_1 + \frac{T}{2} \ l_{2,i}$
- “Average” current? Square current?
We want to maximize:

- “Total” current: \( \frac{T}{2} I_1 + \frac{T}{2} I_2 \)

- “Average” current? Square current?

\( F(I_1, I_2) \): a monotonely increasing function of \( I_1, I_2 \)
Adaptive control

Simplification:
$R$ is a discrete random variable. $P(R = r_i) = p_i, i = 1, 2, \ldots, n$ (known).

1. At time $\tau = 0$, we compute values $l_1$, and $l_{2,i}$ for $i = 1, 2, \ldots, n$. These values are used as follows:
2. For all $t \in [0, \tau/2]$, we set $l(t) = l_1$.
3. At time $\tau/2$, we observe the value of $R$. Assuming $R = r_i$, then for all $t \in [\tau/2, \tau]$, we set $l(t) = l_{2,i}$.

Goals:

(a) $P(H(\tau) > k) < \epsilon$. $k$ smaller than critical temperature
(b) $l_1 \leq L(\tau/2)$.
(c) Maximize:

$$\sum_{i=1}^{n} F(l_1, l_{2,i})p_i$$
\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} F(l_1, l_{2,i})p_i \\
\text{s.t.} & \quad P(H(\tau) > k) \leq \epsilon
\end{align*}
\]
\[
\text{max } \sum_{i=1}^{n} F(I_1, l_2, i) p_i \\
\text{s.t. } P(H(\tau) > k) \leq \epsilon \\
H(\tau) \leq u
\]
max \sum_{i=1}^{n} F(I_1, I_2, i) p_i \\
s.t. \quad P(H(\tau) > k) \leq \epsilon \\
\quad H(\tau) \leq u \quad (k < u < \text{critical temp})
\[
\max \sum_{i=1}^{n} F(I_1, I_2, i)p_i \\
\text{s.t.} \quad P(H(\tau) > k) \leq \epsilon \\
H(\tau) \leq u \quad (k < u < \text{critical temp}) \\
l_1 \leq L(\tau/2, k)
\]
max \sum_{i=1}^{n} F(l_1, l_{2,i}) \rho_i \\
\text{s.t.} \quad P(H(\tau) > k) \leq \epsilon \\
H(\tau) \leq u \quad (k < u < \text{critical temp}) \\
l_1 \leq L(\tau/2, k) \\
\text{other constraints.}
\[
\max \sum_{i=1}^{n} F(l_1, l_2, i) p_i \\
\text{s.t. } P(H(\tau) > k) \leq \epsilon \\
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\text{other constraints.}
\]

Recall:

\[
H(\tau) = \int_0^{\tau} e^{-\nu(\tau-s)} \alpha l^2(s) ds + R(1 - e^{-\nu\tau}) + Ce^{-\nu\tau},
\]
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\[
= v_1 l_1^2 + v_2 l_2^2(i) + r_i(1 - e^{-\nu \tau}) + Ce^{-\nu \tau} \quad \text{in state } i
\]
\[
\begin{align*}
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\]

So chance constraint is of the form:

\[
\sum_{i=1}^{n} \mathbb{I}\{v_1 l_1^2 + v_2 l_2^2(i) > u - r_i(1 - e^{-\nu \tau}) - Ce^{-\nu \tau}\} p_i \leq \epsilon.
\]
\[
\text{max} \sum_{i=1}^{n} F(l_1, l_{2,i}) p_i \\
\text{s.t.} \quad P(H(\tau) > k) \leq \epsilon \\
H(\tau) \leq u \quad (k < u < \text{critical temp}) \\
l_1 \leq L(\tau/2, k) \\
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Recall:

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H(\tau) = \int_0^{\tau} e^{-\nu(\tau-s)} \alpha l^2(s) ds + R(1 - e^{-\nu \tau}) + C e^{-\nu \tau},
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So chance constraint s of the form:

\[
\sum_{i=1}^{n} \left\{ v_1 \ l_1^2 + v_2 \ l_2^2(i) > u - r_i (1 - e^{-\nu \tau}) - C e^{-\nu \tau} \right\} p_i \leq \epsilon.
\]
\[
\begin{align*}
\text{max } & \quad \sum_{i=1}^{n} \tilde{F}(z_1, z_2(i)) \ p_i \\
\text{s.t. } & \quad \sum_{i=1}^{n} \mathbb{I}\{z_1 + z_2(i) > w_i\} p_i \leq \epsilon \\
& \quad z_1 + z_2(i) \leq u_i \quad (w_i < u_i) \\
& \quad z_1 \leq \bar{k} \\
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\end{align*}
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\end{align*}
\]

**Lemma:** At optimality,

\[ z_1 + z_2(i) = w_i \text{ or } u_i, \quad \text{all } i \]
\[ \max \sum_{i=1}^{n} \tilde{F}(z_1, z_2(i)) \, p_i \]

s.t. \[ \sum_{i=1}^{n} \mathbb{I}\{z_1 + z_2(i) > w_i\} p_i \leq \epsilon \]
\[ z_1 + z_2(i) \leq u_i \quad (w_i < u_i) \]
\[ z_1 \leq \bar{k} \]
other constraints.

**Lemma:** At optimality,
\[ z_1 + z_2(i) = w_i \text{ or } u_i, \quad \text{all } i \]

→ Use **binary** variable

\[ y_i = \begin{cases} 0 & \text{when } z_1 + z_2(i) = w_i \\ 1 & \text{when } z_1 + z_2(i) = u_i \end{cases} \]
Continuous knapsack problem

$$\max \sum_{i=1}^{n} \tilde{F}(z_i, w_i - z_i)p_i(1 - y_i) + \tilde{F}(z_i, u_i - z_i)p_i y_i$$

s.t. $$\sum_{i=1}^{n} u_i p_i y_i \leq \epsilon$$

$$0 \leq z_1 \leq \bar{k}$$

$$y_i = 0 \text{ or } 1, \text{ all } i.$$
Continuous knapsack problem

$$\max_{z_1 \in [0, \bar{k}]} M(z_1)$$
Continuous knapsack problem

\[
\max_{z_1 \in [0, \bar{k}]} M(z_1) \\
M(z_1) \triangleq \sum_{i=1}^{n} \tilde{F}(z_1, w_i - z_i)p_i(1 - y_i) + \tilde{F}(z_1, u_i - z_i)p_i y_i \\
s.t. \sum_{i=1}^{n} u_i p_i y_i \leq \epsilon \\
y_i = 0 \text{ or } 1, \quad \text{all } i.
\]
Continuous knapsack problem

\[
\begin{align*}
\max_{z_1 \in [0, \bar{k}]} & \quad M(z_1) \\
M(z_1) & \equiv \sum_{i=1}^{n} \tilde{F}(z_1, w_i - z_i) p_i (1 - y_i) + \tilde{F}(z_1, u_i - z_i) p_i y_i \\
\text{s.t.} & \quad \sum_{i=1}^{n} u_i p_i y_i \leq \epsilon \\
& \quad y_i = 0 \text{ or } 1, \quad \text{all } i.
\end{align*}
\]

Practicable!
Identifying Risky Contingencies of Transmission Systems

(Joint with S. Harnett, T. Kim and S. Wright)

- N - 1 criterion widely used. But is it enough?
- How about N - K, for K “larger”? Everybody knows that:
  - It is too slow. A very difficult combinatorial problem.

<table>
<thead>
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</table>

- It is too conservative. It is not conservative enough.
  (T. Boston) during Hurricane Sandy, N - 142 was observed.
- Perhaps N - K does not necessarily capture all interesting events?
Example: August 14 2003

U.S. - Canada report on blackout:
“Because it had been hot for several days in the Cleveland-Akron area, more air conditioners were running to overcome the persistent heat, and consuming relatively high levels of reactive power – further straining the area’s limited reactive generation capabilities.”

- A **system-wide** condition that impedes the system
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- Not a cause, but a contributor
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- A system-wide condition that impedes the system
- Not a cause, but a contributor
- Look for events that combine both effects?
**A continuous interdiction model**

- A fictitious adversary is trying to interdict the transmission system.

- This adversary negatively alters the physical parameters of equipment, e.g. transmission lines, so as to impede transmission.

- The adversary has a budget available (both system-wide and per-line).

  - On line $km$, reactance $x_{km}$ increased to $(1 + \lambda_{km})x_{km}$
A blast from the past: B. and Verma 2007

- **DC approximation** to power flows.
- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

\[
\max_{x, \theta} \max_{km} \left\{ \frac{|\theta_k - \theta_m|}{u_{km} x_{km}} \right\} = \max_{km} \frac{\text{flow on line } km}{\text{limit of line } km}
\]

s.t. \( B_x \theta = d \)

\( x \) within budget

- Continuous, but non-smooth problem.
A blast from the past: B. and Verma, 2007

- **DC approximation** to power flows.
- Adversary **increases reactances** of lines.
- **Limit** on total percentage-increase of reactances, and on per-line increase.
- Adversary maximizes the maximum **line overload**:

\[
\begin{align*}
\max_{x,\theta,\alpha} & \quad \sum_{km} (\alpha_k^+ - \alpha_k^-) \left( \frac{\theta_k - \theta_m}{u_{km}} \right) x_{km} \\
\text{s.t.} & \quad B_x \theta = d \\
\text{and} & \quad x \text{ within budget} \\
& \quad \sum_{km} (\alpha_k^+ + \alpha_k^-) = 1, \quad \alpha^+, \alpha^- \geq 0.
\end{align*}
\]

- Continuous, smooth, **nonconvex**.
And what happens?

- Efficient computation of gradient and Hessian of objective
- Local optimization algorithm implemented using LOQO
- Algorithm scales well (2007): CPU times of $\sim 1$ hour for studying systems with thousands of lines.
- Optimal * attack concentrated on a handful of lines
- Plus system-wide attack impacting many lines
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</tbody>
</table>

“single” = max multiplicative increase of a line’s reactance
“total” = max total multiplicative increase of line reactances
Today: the AC power flows setting

Adversary increases impedances, so as to **maximize**:
- Phase angle differences across ends of a lines
- Voltage deviations (loss)
- Lost load following recourse actions

Generically:

\[
\max \ F(x) \\
\text{s.t.} \quad x \in B
\]

- \(x\) = impedances, \(B\) = budget constraints
- \(F(x)\) = measure of phase angle differences, voltage loss, load loss
- Challenge: \(F(x)\) is implicitly defined (bilevel optimization problem)
Voltage attack on 2383-bus Polish

“Double the reactance of at most three lines”

Primarily 4 lines interdicted