Robust Models of Epidemics, and Emergency Resource Allocation

Daniel Bienstock, joint with A. Cecilia Zenteno

Columbia University

USC Epstein, February 2013
Motivation

- Virus mutates continuously $\rightarrow$ epidemic
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- How to combat its impact?
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  - Mortality and morbidity $\rightarrow$
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How to combat its impact?

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  - Public health interventions:
    - Vaccine and antivirals
    - Non-pharmaceutical interventions
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  - Mortality and morbidity $\rightarrow$
    - Public health interventions:
      - Vaccine and antivirals
      - Non-pharmaceutical interventions
- Workforce absenteeism
Motivation

Objective: Counteract impact of epidemic-related absenteeism on operation of critical infrastructure.
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- Energy plants
- Water plants
- Supply chains
- Hospitals and clinics
What to do?

- WHO, CDC, HHS - preparedness recommendations
What to do?

- Comparative analysis of national pandemic influenza preparedness plans

- WHO, CDC, HHS - preparedness recommendations

- Absenteeism → surge staff
What to do?

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- Volunteer networks and DB
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- WHO, CDC, HHS - preparedness recommendations
- Absenteeism → surge staff
  - Volunteer networks and DB
  - Students (health services)
  - Recent retirees
- Planning horizon - fully preplanned
- When and how many?
Agenda

1. Disease Modeling
2. Hiring Restrictions & Implementation
3. System Utilization Measure
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2. Hiring Restrictions & Implementation
3. System Utilization Measure
1. A model for influenza

- **SEIR model**
  - Deterministic
  - Spread of the disease in large populations
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- **Individuals → compartments**

  \( S \)  Susceptible

  \( E \)  Exposed or latent

  \( I \)  Infectious

  \( R \)  Removed

\[
\lambda \beta p \rightarrow \mu_E \rightarrow \mu_{RR} \]

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  E  Exposed or latent
  I  Infectious
  R  Removed

\[ \begin{align*}
\lambda & \quad \text{avg contacts} \\
\beta & \quad \mathbb{P}\{\text{contact I}\} \\
p & \quad \mathbb{P}\{\text{contagion}\} \\
\mu_E & \quad \text{Incubation rate} \\
\mu_{RR} & \quad \text{Removal rate}
\end{align*} \]

Diagram of the SEIR model with transitions:
- \( S \rightarrow E \) via \( \lambda \beta p \)
- \( E \rightarrow I \) via \( \mu_E \)
- \( I \rightarrow R \) via \( \mu_{RR} \)
1. A model for influenza

- **SEIR model**
  - Deterministic
  - Spread of the disease in large populations

- Individuals $\rightarrow$ compartments

  - $S$: Susceptible
  - $E$: Exposed or latent
  - $I$: Infectious
  - $R$: Removed

  \[
  \lambda \beta p \rightarrow \mu_E \rightarrow \mu_{RR} \rightarrow \text{avg contacts}
  \]
  \[
  \beta \mathbb{P}\{\text{contact I}\} = I/N
  \]
  \[
  p \mathbb{P}\{\text{contagion}\}
  \]
  \[
  \mu_E \text{ Incubation rate}
  \]
  \[
  \mu_{RR} \text{ Removal rate}
  \]
Workers

Keep track of absenteeism → separate accounting of workers.
Keep track of absenteeism $\rightarrow$ **separate** accounting of workers.

\[
\begin{align*}
& S_1 \xrightarrow{\lambda_1 \beta p} E_1 \xrightarrow{\mu_{E_1}} I_1 \xrightarrow{\mu_{RR_1}} R_1 \rightarrow \text{General population} \\
& S_2 \xrightarrow{\lambda_2 \beta p} E_2 \xrightarrow{\mu_{E_2}} I_2 \xrightarrow{\mu_{RR_2}} R_2 \rightarrow \text{Workers}
\end{align*}
\]
Keep track of absenteeism → separate accounting of workers.

$$\beta = \frac{\lambda_1 I^1 + \lambda_2 I^2}{\lambda_1 N^1 + \lambda_2 N^2}$$
Motivation Model Robust Optimization

Discrete time SEIR model

Model for subgroup $j$ transition $t \rightarrow t + 1$:

\[
\begin{align*}
S_{t+1}^j &= S_t^j e^{-\lambda_j^* \beta_t^* p} \\
E_{t+1}^j &= E_t^j e^{-\mu E_j} + S_t^j (1 - e^{-\lambda_j^* \beta_t^* p}) \\
I_{t+1}^j &= I_t^j e^{-\mu_{RR}^j} + E_t^j (1 - e^{-\mu E_j}) \\
R_{t+1}^j &= R_t^j + I_t^j (1 - e^{-\mu_{RR}^j}).
\end{align*}
\]

[LJS Allen et al, 1991; Larson, 2007]
Discrete time SEIR model

Model for subgroup $j$ transition $t \rightarrow t + 1$:

\[
\begin{align*}
S^j_{t+1} &= S^j_t e^{-\lambda_j \beta_t p} \\
E^j_{t+1} &= E^j_t e^{-\mu E_j} + S^j_t (1 - e^{-\lambda_j \beta_t p}) \\
I^j_{t+1} &= I^j_t f_j e^{-\mu RR_j} + E^j_t (1 - e^{-\mu E_j}) \\
R^j_{t+1} &= R^j_t + I^j_t (1 - e^{-\mu RR_j}).
\end{align*}
\]

[LJS Allen et al, 1991; Larson, 2007]
Non-homogeneous contact

- It is likely that social contacts will change during epidemic
Non-homogeneous contact

- It is likely that social contacts will change during epidemic

\[ \chi^j_t = \Lambda^j \frac{S^j_t + E^j_t + R^j_t}{N^j_t} \]

[Allen et al, 1991]
An inconvenient fact

- SEIR models $\rightarrow$ uncertain many parameters
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- New mutation - at best, noisy estimations
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- Incubation and recovery rates ($\mu_E, \mu_R$) are “easy”
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An inconvenient fact

- SEIR models → uncertain many parameters
- New mutation - at best, noisy estimations
- Incubation and recovery rates ($\mu_E, \mu_{RR}$) are “easy”
- Contagion rate $\lambda \beta p$?
- Focus uncertainty on probability of contagion $p$
Planning under uncertainty

Leave SEIR parameters fixed, except probability of contagion, $p$. 

![Graph showing workforce shortage over time with different probabilities of contagion](image-url)

- $p = 0.03$
- $p = 0.02$

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Planning under uncertainty

Leave SEIR parameters fixed, except probability of contagion, $p$. 

![Diagram showing workforce shortage over time with varying $p$ values]
Planning under uncertainty

Leave SEIR parameters fixed, except probability of contagion, $p$. 
2. Implementing a strategy

- Bringing in surge staff - restrictions
2. Implementing a strategy

- Bringing in surge staff - restrictions
  - Limited availability
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- Bringing in surge staff - restrictions
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  - Quantity
2. Implementing a strategy

- Bringing in surge staff - restrictions
  - Limited availability
    - Quantity
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  - Can also get sick
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- When is the surge strategy rolled out?
When does the surge commence?

- Epidemic declared when growth rate of infectious > threshold
When does the surge commence?

- **Epidemic declared** when growth rate of infectious > threshold

- **Assumption**: Deploy strategy only after epidemic is declared
When does the surge commence?

- **Epidemic declared** when growth rate of infectious > threshold

- **Assumption**: Deploy strategy only after epidemic is declared

- **Assumption**: Epidemic is *correctly* declared
A technical detail

- Epidemics with different “p” declared at different times

![Graph showing workforce shortage over time for different values of p](image)
A technical detail

- the planner’s perspective:
A technical detail

- the planner’s perspective:
3. Quantifying the impact - Utilization measures

Total “social” cost: sum of per day costs

Two specific settings:

- Min workforce level to operate
  $m$ - threshold

\[
\text{Cost} = \sum_{t} z_t
\]
3. Quantifying the impact - Utilization measures

Total “social” cost: sum of per day costs

Two specific settings:

- Min workforce level to operate \( m \) - threshold

- Queueing theory
  System utilization \( \rho_t \)

\[
\text{Cost} = \sum_{t} z_t
\]

\[
\text{Cost} = \sum_{t} e^{K(\rho_t-1)^+} - 1
\]
3. Quantifying the impact - Utilization measures

Total “social” cost: sum of per day costs

Two specific settings:

- Min workforce level to operate $m$ - threshold
- Queueing theory
  System utilization $\rho_t$

Convex piecewise-linear functions
First Optimization Model

Assumption: Size of surge staff corps is small relative to population; so staff deployment does not alter epidemic

Key modeling variables:

∀ time periods $t' > t$, the quantities of surge staff that

- are first deployed at time $t$, and
- are susceptible, or exposed, or infected at time $t'$
First Optimization Model

\[ V(h|p) := \text{impact of epidemic under strategy } h, \text{ given prob of contagion } p \]
First Optimization Model

- $V(h|p) :=$ impact of epidemic under strategy $h$, given prob of contagion $p$

Robust Optimization Problem

$$V^* = \min_{h \in H} \max_{p \in P} V(h|p)$$

Objective: Strategy resilient against all scenarios

$H \leftarrow$ set of feasible surge strategies
$P \leftarrow$ uncertainty set
Some formulation details

- at time $t$, variable $a_t = \text{total number of available staff}$
  $= \text{original staff, non-infective at time } t \ (\text{known from SEIR model})$
  $+ \text{surge staff, non-infective at time } t$
Some formulation details

- at time $t$, variable $a_t = \text{total number of available staff}$
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  $+ \text{surge staff, non-infective at time } t$

- use SEIR equations to keep track of the latter: \textbf{linear for fixed } p
Discrete time SEIR model applied to surge staff

For each given time \( t' \), track of condition of staff deployed at \( t' \):

- \( h_{t'} \): quantity deployed at \( t' \)
- \( s_{t,t'}^s \): quantity deployed at \( t' \) and susceptible at \( t \),
- \( e_{t,t'}^s \): quantity deployed at \( t' \) and exposed at \( t \),

\[
\begin{align*}
    s_{t',t'}^s &= h_{t'} \\
    s_{t+1,t'}^s &= s_{t,t'}^s e^{-\lambda_s \beta_t p} \\
    e_{t+1,t'}^s &= e_{t,t'}^s e^{-\mu_s} + s_{t,t'}^s (1 - e^{-\lambda_j \beta_t p})
\end{align*}
\]

and for all \( t' \leq t < t' + K \),
Some formulation details

- at time $t$, variable $a_t =$ total number of available staff
  = original staff, non-infective at time $t$ (known from SEIR model)
  + surge staff, non-infective at time $t$

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- Likewise, constraints to keep track of (convex) costs are linear
Example:

- $f(z) = \text{piecewise-linear increasing function of } z$
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- So cost = \( \kappa_t = f((\theta_t - a_t)^+) \)
Example:

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Example:

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- So cost = \( \kappa_t = f((\theta_t - a_t)^+) \)
- So: constraint: \( \Gamma_t \geq \theta_t - a_t \), variable \( \Gamma_t \geq 0 \),
- and constraint: \( \kappa_t \geq s_i \Gamma_t + b_i \), for \( 1 \leq i \leq l_t \) \( (s_i \geq 0 \text{ for all } i) \)
Summary: First Optimization model

\[ V(h|p) : \]
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\[ V(h|p) : (\text{given } p) \text{ can be formulated as an LP} \]
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- \( p \) fixed \( \rightarrow \) we know trajectory of epidemic
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\( V(h|p) : (\text{given } p) \) can be formulated as an LP

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Summary: First Optimization model

\( V(h|p) : \) (given \( p \)) can be formulated as an LP

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Summary: First Optimization model

\[ V(h|p) : \text{(given } p\text{)} \text{ can be formulated as an LP} \]

\[ V(h|p) := \min \quad c^T x \]

\[ \text{s.t.} \quad A_p x = h \quad \text{(SEIR eqs)} \]

\[ C_p x \geq d_p \quad \text{(piecewise-linear approx)} \]

\[ x \geq 0, \quad x \in H \]

\[ x \leftarrow \text{groups } SE(IR) + \text{objective function aux. variables} \]

\[ H \leftarrow \text{set of feasible strategies} \]
Solving the problem

Our problem:

\[ V^* = \min_{h \in H} \max_{p \in P} V(h|p) \]

(An infinite LP.) How to solve?
Solving the problem

Our problem:

\[ V^* = \min_{h \in H} \max_{p \in P} V(h \mid p) \]

(An infinite LP.) How to solve?

(Today) Approximate model: use finite \( Q \subset P \)

\[ V^* \approx \min \quad c^T x \]

s.t. \[ A_p x = h, \quad \forall p \in Q \]
\[ C_p x \geq d_p, \quad \forall p \in Q \]
\[ h \in H \]
\[ x \geq 0 \]

Example: \( P = [0.01, 0.013], Q = \) subset of \( P \) at integral multiples of 0.0001
Theoretical justification

Suppose we know that $p \geq p_0 > 0$.

For each $\epsilon > 0$ small enough there is a $\delta = O(\epsilon)$ s.t.:

If $|p - p'| < \delta$ then $V(h|p') \leq (1 + \epsilon)V(h|p)$ for any $h$
Benders’ Decomposition

- Generalized Benders’ Decomposition
Benders’ Decomposition

- Generalized Benders’ Decomposition

Optimization problem

\[
V^* = \min \max_{h \in H} V(h|p) \\
= \min_{z, h \in H} z \\
\text{s.t. } z \geq V(h|p) \quad \forall \ p \in P
\]
Benders’ Decomposition

- Generalized Benders’ Decomposition
- Idea: replace $V(h|p)$ by cuts obtained from the dual

Optimization problem

$$V^* = \min_{h \in H} \max_{p \in P} V(h|p)$$

$$= \min_{z,h \in H} z$$

s.t. $z \geq \alpha_p^T h + \pi_p^T d_p$, (dual $\forall p \in P$)
Benders’ Decomposition

- Generalized Benders’ Decomposition
- Idea: replace $V(h|p)$ by cuts obtained from the dual

**Approximation**

$$V^* \approx \min_{h \in H} \max_{p \in P} V(h|p)$$

$$= \min_{z, h \in H} z$$

s.t. $z \geq \alpha_p^T h + \pi_p^T d_p$, (dual $\forall p \in Q$)

$Q$ is a **relatively small** subset of $P$ - so separation problem is fast
Basic Algorithm

Iterate:

1. Solve Master Problem; let $\hat{h}$ be the computed surge and $\hat{z}$ be the estimate of its worst-case cost.

2. **Sample:** compute the worst-case data realization for $\hat{h}$.

3. If the cost of $\hat{h}$ under this resolution is at most $\hat{z}$, **STOP**.

4. Otherwise, add to the master a duality cut violated by $\hat{h}$, $\hat{z}$, and **goto 1**.
Algorithmic enhancements

- “Powers of two” approximation to finite grid
Algorithmic enhancements

- “Powers of two” approximation to finite grid
- “Pre-Benders” cuts
Alternative Optimization Problem 1

- Intervals or tranches $l_1, \ldots, l_1$ of $[0, 1]$; a time period $J$
- At time $= 1$, it is known that $p \in l_1$.
- At time $J$ there is a switch. For $t \geq J$, $p \in l_h$ (known at $t = J$)

Decision maker:
- Rolls out a surge at $t = 1$ that covers periods $1 \leq t < J$,
- At $t = 1$ announces $m$ surge plans to cover periods $J \leq t \leq T$
- At time $= J$, switches to one of the announced plans
Alternative Optimization Problem 2

- There is a known interval $I$ such that $p_t \in I$ for all $t$.
- At time $t$, $p_t = \mu + \delta_t$, and is observed.
- Here $\mu =$ midpoint of $I$, and $\delta_t =$ zero mean stochastic, small.

Decision maker:

- At time 1, announces “expected” surge quantities $h_t$ for all $t$, and a multiplier $\lambda \geq 0$.
- At time $t$, corrects $h_t$ by $\lambda \frac{\sum_{j<t} (p_j - \mu)}{t}$.
- (up to a maximum allowable)
More general uncertainty sets

- Flexible algorithm - more general uncertainty sets
More general uncertainty sets

- Flexible algorithm - more general uncertainty sets
- Sudden weather changes [Lowen et al, 2007]
- Public Health measures could change course of epidemic

Daily cumulative incidence
Flu epidemic - San Francisco

SanFran
\( p_{cont} = 0.18 \)
\( p_{cont} = 0.2 \)
\( p_{cont} = 0.194 \)
More general uncertainty sets

- Flexible algorithm - more general uncertainty sets
- Sudden weather changes [Lowen et al, 2007]
- Public Health measures could change course of epidemic
- Analyze the impact of multiple values of $p$ during 1 epidemic

Daily cumulative incidence
Flu epidemic - San Francisco

SanFran
$p_{cont} = 0.18$
$p_{cont} = 0.2$
$p_{cont} = 0.194$
Numerical example

Demography

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<th>High Risk Population</th>
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<tbody>
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- **Uncertainty set**
  - \( P = [0.01, 0.012] \times [0.0125, 0.0135] \)
  - \( p \) can change in days \{140, ..., 160\}
Numerical example

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- $P = [0.01, 0.012] \times [0.0125, 0.0135]$
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Pool of surge staff

- Up to 3,000 staff
- Stay up to 1 week
Numerical example

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Pool of surge staff
- Up to 3,000 staff
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Social Contact Model
- Non-homogeneous contact
- Contact rate decreases 30% when epidemic is declared
Comparisons:

- **Do nothing at all** (how bad is the "worst" epidemic?)
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- **Naïve Worst-Case** planning: prepare for the data realization that is most expensive in the “do nothing” case
Comparisons:

- **Do nothing at all** (how bad is the “worst” epidemic?)

- **Naïve Worst-Case** planning: prepare for the data realization that is most expensive in the “do nothing” case

- **The robust strategy**
No surge staff deployment

Most costly data realization: \((p_1, p_2, d) = (0.0109, 0.0135, 140)\)
Cost: 4.58
Same data realization, but using Robust Strategy

\[(p_1, p_2, d) = (0.0109, 0.0135, 140)\]

Cost: 0.0495
.. and using Naïve Worst-Case Strategy

\[(p_1, p_2, d) = (0.0109, 0.0135, 140),\]
.. and using Naïve Worst-Case Strategy

\[(\rho_1, \rho_2, d) = (0.0109, 0.0135, 140), \text{ Cost: 0}\]
Worst scenario, *when played against* the robust strategy: 
\((p_1, p_2, d) = (0.01168, 0.0135, 140)\)
Worst scenario, when played against the robust strategy: $(p_1, p_2, d) = (0.01168, 0.0135, 140)$

Using this data realization, “do nothing” cost: 1.69
Same data: \((p_1, p_2, d) = (0.01168, 0.0135, 140)\)

... against the Robust Strategy, Cost: 0.05
Same data: \((p_1, p_2, d) = (0.01168, 0.0135, 140)\)
... against “Naïve Worst-Case” Strategy
Cost: 0.69
Example - Comparing strategies

Scenario I

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Example - Comparing strategies

Scenario II

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# Example - Comparing strategies

<table>
<thead>
<tr>
<th></th>
<th>No Intervention</th>
<th>Robust Strategy</th>
<th>Worst-Case Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No intervention:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>4.581</td>
<td>0.050</td>
<td>0.000</td>
</tr>
<tr>
<td>worst tuple</td>
<td>1.002</td>
<td>1.048</td>
<td>1.000</td>
</tr>
<tr>
<td>(0.01092, 0.0135, 140)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical days (ρ &gt; 1)</td>
<td>28</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td><strong>Robust Strategy:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>1.694</td>
<td>0.052</td>
<td>0.686</td>
</tr>
<tr>
<td>worst tuple</td>
<td>1.024</td>
<td>1.003</td>
<td>1.017</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical days (ρ &gt; 1)</td>
<td>21</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td><strong>Worst-case Strategy:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
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<td>0.050</td>
<td>0.710</td>
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<td>1.018</td>
</tr>
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</tr>
<tr>
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<td>8</td>
<td>13</td>
</tr>
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</table>
Example - Comparing strategies

Take-away: Planning against most expensive scenario is not enough!

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Cost (0.01)</th>
<th>Cost (0.05)</th>
<th>Cost (0.10)</th>
</tr>
</thead>
<tbody>
<tr>
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</table>
Example - Out-of-sample Analysis

- Uncertainty set
  - $P = [0.01, 0.012] \times [0.0125, 0.0135]$
  - $p$ can change in days \{140, ..., 160\}

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Example - Out-of-sample Analysis

- Uncertainty set
  - $P = [0.01, 0.012] \times [0.0125, 0.0135]$
  - $p$ can change in days $\{140, \ldots, 160\}$
Example - Out-of-sample Analysis

- Uncertainty set
  
  - $P = [0.01, 0.012] \times [0.0125, 0.0135]$
  
  - $p$ can change in days $\{140, \ldots, 160\}$
  
  - Worst case: $(p_1, p_2, d) = (0.01168, 0.0135, 140)$

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Original</th>
<th>Hypothetical 1</th>
<th>Hypothetical 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0.01168</td>
<td>0.01168</td>
<td>0.01168</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.0135</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>day epidemic is declared</td>
<td>113</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>day deployment starts</td>
<td>140</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>day $p$ changes</td>
<td>140</td>
<td>150 155 160 165-</td>
<td>150 155 160 165</td>
</tr>
</tbody>
</table>

| cost Robust Policy           | 0.0508   | 0.3268 0.07295 0 | 2.1737 1.4068 0.6282 0.0294 |
| cost No Intervention         | 4.58     | 1.609 0.762 0.087 0 | 4.133 2.669 1.243 0.146 |
Example - Numeric Performance

- 350 days
- max-cost tuple: grid search
- VBA (UI) + C (SEIR) + AMPL (Gurobi solver)
Example - Numeric Performance

- 350 days
- max-cost tuple: grid search
- VBA (UI) + C (SEIR) + AMPL (Gurobi solver)
- ~ 175 iterations
- 5% duality gap
- ~ 15min CPU time
Example - Numeric Performance

- 350 days
- max-cost tuple: grid search
- VBA (UI) + C (SEIR) + AMPL (Gurobi solver)

Enhancement:
- 8 pre-Benders’ iterations + 1 Benders’ cut
- Duality gap 0.0052%
- < 1 min CPU time