COLUMBIA BUSINESS SCHOOL

B6015: Decision Models
Spring 2002
Sample Questions from Past Final Exams

Here are two sample questions, one optimization and one simulation, from past final exams.

1. Briton Paper Company

Susan Jetson is a transportation planner for Briton Paper Company. Ms. Jetson needs to assign truck carriers for shipments of finished paper. Truckloads of paper need to be delivered from Briton's paper mill to each of 5 destinations labelled *a* through *e*. The data for the problem is given in Table 1. Two truckloads of paper must be delivered to location *a*, four truckloads to location *b*, etc…. Briton uses four carriers and each carrier has a limited number of available trucks. The rate table in Table 2 indicates the amount that each carrier charges per truckload, in $ per mile. If the resulting charge is less than the carrier's minimum charge, the carrier must be paid at least the minimum charge for the delivery of that truckload. For example, if Ms. Jetson decides to use carrier 3 to ship one truckload to location *a* the charge is $624. If she uses carrier 3 to ship one truckload to location *e* the charge is $500. (All trucks must leave this morning. Because of the travel time involved, a truck can ship to a single destination only.) The data for the minimum charges, and the number of trucks available from each carrier appear in Table 3.

Table 1. *Destinations, Distances and Requirements*

	a	*b*	*c*	*d*	*e*
Distance from mill (miles)	600	1500	1200	3000	150
Requirements (truckloads)	2	4	3	5	5

Table 2. *Rate table* ($ per mile per truckload)

	a	*b*	*c*	*d*	*e*
Carrier 1	$1.48	$1.45	$1.47	$1.22	$1.26
Carrier 2	$1.76	$1.46	$1.79	$1.63	$1.43
Carrier 3	$1.04	$1.37	$1.26	$1.17	$1.18
Carrier 4	$1.38	$1.45	$1.59	$1.44	$1.56

Table 3. *Carrier Information*

Carrier #	*1*	*2*	*3*	*4*
Trucks available	5	6	6	5
Minimum Charge (per truckload)	$400	$500	$500	$500

In the past Ms. Jetson has made carrier assignments by hand, relying on her experience as a transportation planner for the last several years. Today, Ms. Jetson set up a spreadsheet to assist her planning. She used a spreadsheet optimizer to obtain the results shown in Figure 1. (Some information in the cost table has been hidden from view.) Figure 2 shows the answer report corresponding to the optimized spreadsheet.

Figure 1. The Briton Spreadsheet

BRITON.XLS **Briton Paper Company**

Carrier	Number of Trucks Available	Minimum Charge
1	5	$400
2	6	$500
3	6	$500
4	5	$500

Total Cost: $ 34,957

Destinations	a	b	c	d	e
Distance to mill	600	1,500	1,200	3,000	150

Assignment of trucks to destinations

	a	b	c	d	e	Total		Number of Trucks Available
Carrier 1	0		0	2	3	5	<=	5
Carrier 2							<=	6
Carrier 3	0		3	3	0	6	<=	6
Carrier 4	2		0	0	0	5	<=	5
Total	2		3	5	3			
	=	=	=	=	=			
Requirement	2	4	3	5	5			

Cost Table

	a	b	c	d	e
Carrier 1					
Carrier 2	$1,056				
Carrier 3	$624				$500
Carrier 4					

Rate Table ($ per mile)

	a	b	c	d	e
Carrier 1	$1.48	$1.45	$1.47	$1.22	$1.26
Carrier 2	$1.76	$1.46	$1.79	$1.63	$1.43
Carrier 3	$1.04	$1.37	$1.26	$1.17	$1.18
Carrier 4	$1.38	$1.45	$1.59	$1.44	$1.56

Figure 2. Sensitivity Report for Briton

Microsoft Excel 8.0a Sensitivity Report
Worksheet: [briton.xls]Briton
Report Created: 2/20/1999 12:44:32 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C16	Carrier 1 a	0	145	888	1E+30	145
D16	Carrier 1 b	0	85	2175	1E+30	85
E16	Carrier 1 c	0	102	1764	1E+30	102
F16	Carrier 1 d	2	0	3660	102	31
G16	Carrier 1 e	3	0	400	31	161
C17	Carrier 2 a	0	213	1056	1E+30	213
D17	Carrier 2 b	1	0	2190	31	15
E17	Carrier 2 c	0	386	2148	1E+30	386
F17	Carrier 2 d	0	1,130	4890	1E+30	1130
G17	Carrier 2 e	2	0	500	15	31
C18	Carrier 3 a	0	31	624	1E+30	31
D18	Carrier 3 b	0	115	2055	1E+30	115
E18	Carrier 3 c	3	0	1512	102	1E+30
F18	Carrier 3 d	3	0	3510	31	102
G18	Carrier 3 e	0	250	500	1E+30	250
C19	Carrier 4 a	2	0	828	31	1E+30
D19	Carrier 4 b	3	0	2175	15	31
E19	Carrier 4 c	0	161	1908	1E+30	161
F19	Carrier 4 d	0	575	4320	1E+30	575
G19	Carrier 4 e	0	15	500	1E+30	15

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
C20	Total a	2	843	2	3	1
D20	Total b	4	2,190	4	3	1
E20	Total c	3	1,762	3	3	2
F20	Total d	5	3,760	5	3	2
G20	Total e	5	500	5	3	2
H16	Carrier 1 Total	5	(100)	5	2	3
H17	Carrier 2 Total	3	0	6	1E+30	3
H18	Carrier 3 Total	6	(250)	6	2	3
H19	Carrier 4 Total	5	(15)	5	1	3

a) In the spreadsheet BRITON.XLS in Figure 1, there are some cells that have been hidden from view. Fill in the blanks:

Formula in cell C26 _____

Formula in cell G6 _____

Formula in cell H17 _____

Formula in cell D20 _____

b) Write a decision model to optimize the selection of carriers for the paper mill. Define all decision variables. You can use spreadsheet notation, =SUM, =SUMPRODUCT, =MAX, =IF, etc., but if a cell contains a formula, write the formula and not the cell number.

Objective function (Explain your answer, do not just indicate the cells):

Decision variables (Explain your answer, do not just indicate the cells):

Constraints (Explain your answer, do not just indicate the cells):

You should not use the computer to answer parts d, e and f. If there is some question that cannot be answered with the information given in the spreadsheet (Figure 1) and the sensitivity report (Figure 2) write down "Answer cannot be determined from the information given". Answers based on models solved on your computer will not be considered valid.

c) Suppose that Ms. Jetson could negotiate with one carrier to provide an additional truck. An additional truck from which carrier would provide the biggest reduction in cost? (circle one)

Carrier 1 Carrier 2 Carrier 3 Carrier 4

Explain.

d) Ms. Jetson just received new information that 2 extra truckloads of paper are needed at destination *a*. What is the new optimal cost?

Explain.

e) Carrier 2 has decided to change its rate agreement with Briton paper. Carrier 2 will now charge only $1.46 per mile for going to destination *a*. Will this new rate affect the result of the optimized spreadsheet? (circle one)

YES NO

Explain.

g) Briton paper has agreed to give carrier 2 at least 25% of its business each day (i.e., 25% of the total shipping cost). Modify the formulation from part (b) to accommodate this agreement. Specify the modification in a linear fashion. (Do not solve the model.)

You can use your computer to answer the next question.

h) Ms. Jetson now learns that 2 extra truckloads of paper are needed at destination *a* and only 2 truckloads are needed at destination *c*. What is the new optimal shipping plan and the new optimal cost?

New optimal plan:

	A	*b*	*c*	*d*	*e*	*Total*
Carrier 1						
Carrier 2						
Carrier 3						
Carrier 4						
Total						

Cost table used (cells B28:F31 in your spreadsheet):

	a	*b*	*c*	*d*	*e*
Carrier 1					
Carrier 2					
Carrier 3					
Carrier 4					

New optimal cost:

_____

2. *NiftyCalc. Inc*

Janice Cray has written a radical new spreadsheet program called NiftyCalc. Ms. Cray needs capital to market and distribute the program, and she is raising money by issuing floating-rate junk bonds. Each junk bond sells for $100 today. If the company succeeds, at the end of the year, the owner of the bond will receive the $100 principal plus an interest payment based on a rate of five times the prime rate at that time. If the company does not succeed, at the end of the year, the owner of each bond will receive an amount of money based on the liquidation value of the company (which is made specific below).

Investment bankers have estimated that the prime rate one year from now follows a normal distribution with a mean of 12% and a standard deviation of 3%. They also estimate that the company will succeed with probability 0.55, will be a marginal failure with probability 0.25, and a total failure with probability 0.20. If the company is a marginal failure, the liquidation value of the company will be such that the owner of the bond receives $70. If the company is a total failure, the liquidation value of the company will be such that the owner of the bond receives $30. The prime rate and the outcome of the company (success, marginal failure or total failure) are independent events.

Table 1. *Uniform Random Numbers (between 0 and 1)*

0.10
0.74
0.53
0.95
0.33
0.19

Table 2. *Normal Random Numbers (mean=12%, SD=3%)*

10.95%
13.05%
9.00%
14.79%
13.80%
18.00%

To estimate the expected return on the junk bond, we will use simulation.

a) Explain how to simulate the cash flow from the bond at the end of the year using the random numbers from Table 1 and/or Table 2. Manually simulate the cash flows for six trials.
E.g. to simulate a flip of a coin using the uniform random numbers in Table 1, you could use the function =IF(X<0.5,Heads,Tails) where X is a uniform random number between 0 and 1. The six simulated coin tosses would then be: Heads, Tails, Tails, Tails, Heads, Heads.

b) Based on your simulation, what do you estimate is the expected return of the bond?

c) Using your simulation results, compute a 90% confidence interval for the bond's expected return. How many trials would you need to simulate to get an estimated expected return within ±1% (at the same 90% confidence level)?

d) Using your simulation results from part a), estimate the probability that the cash flow is at least $105. (Note: you do not need a *z*-table.) Explain the procedure that you are using.

e) Using Crystal Ball, simulate this bond for 500 trials using a random number seed of 123 and Monte-Carlo sampling. What is the expected return? What is the probability that cash flow will exceed $105?