Exploring the Diderot programming language and its applications to the visualization of neural models

Daniel Clark

Department of Electrical Engineering

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Overview

- Review Diderot - uses, continuous fields, high-level simple syntax, parallelism
- Hypothesis: Can we use Diderot to define high-level brain operators (e.g. optical flow)?
- Neural field equation $\stackrel{?}{=}$ Diderot's continuous tensor field
- Initial experiments and limitations of Diderot
- Visualizing the phase plane with line integral convolution (LIC)
- Experimental results and future direction

Diderot's intended use

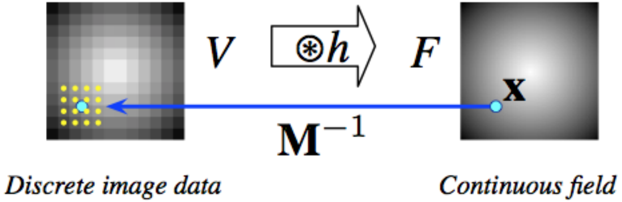
- Domain Specific Language (DSL) - designed for image analysis and visualization
- Image analysis - Extract *quantitative* geometric properties of objects from image data
- Image visualization - use image data in tandem with computer graphics to *qualitatively* describe image properties [1]
- Optimized for algorithms with large number of (mostly) independent computations
- Provides a high-level, simple and direct syntax

Language design and implementation

- Diderot is structured for dealing with data as continuous tensor fields
- Tensor field - scalars (order 0), vectors (order 1), matrices (order 2)
- Formed via convolution of discrete input with "continuous" kernel - `field#k(d)[σ] F = bspln3 ⊛ img`
- `k` - continuous derivatives, `d` - dimensionality, σ - tensor order
- Operators on field include -
  `t = F(pos), ∇F(pos), ∇⊗F(pos);`
  `F = ∇F, ∇⊗F, s*F,` F_1+F_2`;`
  `b = inside(pos,F);`
- `t` is a tensor, which can be manipulated by standard discrete operations, such as - $\bullet$, $\otimes$, $|\ |$, $*$

Field computation

Actual field computation is performed when field is probed -
`t = ∇F, ∇⊗F, F(pos);`



[2]

Field computation

e.g. `t = ∇F(pos);` This performs,

$$\begin{aligned}
\nabla F(x) &= (V \otimes \nabla h)(\mathbf{x}) \\
&= \left(V \otimes \begin{bmatrix} \frac{\partial}{\partial x} h \\ \frac{\partial}{\partial y} h \end{bmatrix} \right) \\
&= \begin{bmatrix} \sum_{i=1-s}^{s} \sum_{j=1-s}^{s} V[\mathbf{n} + < i, j >] h'(\mathbf{f}_x - i) h(\mathbf{f}_y - j) \\ \sum_{i=i-s}^{s} \sum_{j=1-s}^{s} V[\mathbf{n} + < i, j >] h(\mathbf{f}_x - i) h'(\mathbf{f}_y - j) \end{bmatrix},
\end{aligned} \tag{1}$$

where V - discrete input, h - (separable) continuous kernel, $\mathbf{x}$ - field position index, $\mathbf{M}^{-1}$ - space mapping matrix, $\mathbf{n} = \lfloor \mathbf{M}^{-1}\mathbf{x} \rfloor$ discrete mapped point, $\mathbf{f} = \mathbf{M}^{-1}\mathbf{x} - \mathbf{n}$

Program structure

- 3 sections - global definitions, strand, initialization
- Global - define *immutable* variables, inputs, load image data, usually create field
- Strand - computational core of application - run via parameters, init state variables (including output), `update`, `die`, `stabilize` methods
- A given strand will continue to execute its `update` method until the `stabilize` (write to output) or `die` (no write to output) method is called on that strand
- Initialization - define range of strand parameters (e.g. pixel indices)

Parallelism

- Bulk-synchronous parallelism model - super steps (`update` method), each consisting of asynchronous computations (individual `strand`s), executes until all strands `die` or `stabilize`
- Strands organized into (4096 supported) strands per block, followed by barrier synchronization at end of super step
- Parallel C code, OpenCL, CUDA (future)

Toy program

```
// ----- Global defs -----
int imgSizeX = 300;     // how many x pts user wants from field
int imgSizeY = 200;     // how many y pts user wants from field
int stepNum = 25;       // step limit before writing to output
image(2)[] img = load("../data/einstein.nrrd"); // import image
field#1(2)[] F = img  Ł  ctmr; // convolve img with kernel = field

// ----- Strand section -----
strand DEMO (int xi, int yi) {
    real xx = lerp(0.0, 3.0, -0.5, real(xi), real(imgSizeX)-0.5);
    real yy = lerp(0.0, 2.0, -0.5, real(yi), real(imgSizeY)-0.5);
    vec2 pos0 = [xx,yy];
    output real sum = F(pos0);
    int step = 0;
    // --- Update thread with math under condtions ---
    update {
        // Do some fancy math
        vec2 grad =   F (pos);    // take gradient of field and probe it
        sum = grad[0]+grad[1];
        step += 1;
        if (step == stepNum) {
          stabilize;            // write "sum" to output
        }
    }
}

// ----- Initialization section -----
initially [ DEMO(xi, yi) | yi in 0..(imgSizeY-1), xi in 0..(imgSizeX-1) ];
```

Hypotheses

1. We can define high-level operators (analogous to ∇) that will perform encoding of sensory data on a neural circuit.
2. We can represent this circuit as a continuous tensor field.

- Can we use Diderot as a convenient medium for accomplishing this?

Continuous neural fields

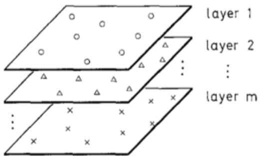
- Utilize maturely developed area of continuous dynamical systems to approximate neural circuitry
- Continuous equations - able to model neural activity as quantities in a continuous field, with functional relationships to sources and sinks *of that field* [3]
- Basic idea: activity of a neuron unit (e.g. average firing rate) in field layer i at point x, time t is

$$Z_i(x,t) = f_i\big(u_i(x,t)\big) \tag{2}$$

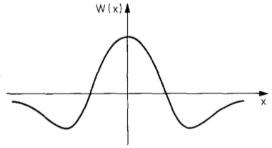
- $u_i(x,t)$ is average membrane potential, f_i is a non-linear activation function [4]

Continuous neural fields (cont'd)

- Model also assumes local excitatory and distant inhibitory inter-connectivity between neurons, described by time-varying weight function
- $w_{ij}(x, y; t)$ - influence activity from neuron at point y in layer j has on neuron at point x in layer i, t time units after firing initiates from neuron y



(a) Neural layers



(b) Weight function

Continuous neural fields (cont'd)

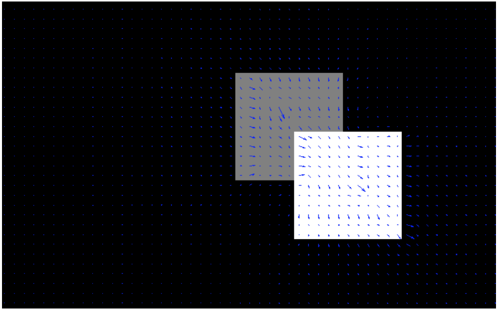
- Canonical continuous field equation [4]:

$$\tau_i \frac{\partial u_i(x,t)}{\partial t} = h_i + \Delta s_i(x,t) - u_i(x,t) + \sum_{j=1}^{m} \int_{\Omega_j} \int_{v} w_{ij}(x,y;t-v) Z_j(y,v)\ dv\ dy \tag{3}$$

- τ_i is the recovery time to steady-state, $\Delta s_i(x,t)$ is change in external stimulus, h_i is average distance to steady-state activity, Ω_j spatial neighborhood of summation on layer j
- Can think of neural field as $N_x \times N_t \times N_j$ continuous tensor field

Optical flow

- If we can define a neural circuit, such as in Eq. (3), what can we do with it?
- Put operation of continuous field into context - specific objective: encode optical flow
- Optical flow - quantification of relative motion in a vision sequence



Optical flow (cont'd)

- In general, three stages in detecting optical flow:
  1. Pre-filter/smooth input data spatiotemporally
  2. Extract relevant features (e.g. gradients)
  3. Weight and integrate features to produce field flow vectors
- Known, in general, as a differential or gradient-based approach

Horn-Schunk method

- Gradient constraint: change along the spatiotemporal path of a point of intensity is zero.

$$\frac{dI(\mathbf{x},t)}{dt} = 0 \;\Rightarrow \nabla_{\mathbf{x}} I(\mathbf{x},t) \cdot \mathbf{v} + \frac{\partial I(\mathbf{x},t)}{\partial t} = 0 \tag{4}$$

$$\nabla_{\mathbf{x}} I(\mathbf{x},t) = \begin{bmatrix} \frac{\partial I(\mathbf{x},t)}{\partial x} \\ \frac{\partial I(\mathbf{x},t)}{\partial y} \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} u \\ v \end{bmatrix} \tag{5}$$

- Horn and Schunk posed this as an optimization problem with smoothness constraint, given by λ [5]

$$\min_{\mathbf{v}} \int_{\Omega} (\nabla I \cdot \mathbf{v} + \frac{\partial I}{\partial t})^2 + \lambda^2 (||\nabla u||_2^2 + ||\nabla v||_2^2) d\mathbf{x} \tag{6}$$

Horn-Schunk method (cont'd)

- HS iterative algorithm:

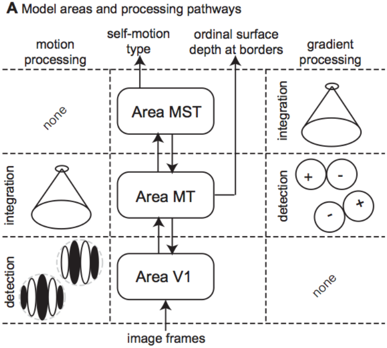
$$u^{k+1} = \bar{u}^k - \frac{I_x\left(I_x\bar{u}^k + I_y\bar{v}^k + I_t\right)}{\alpha^2 + I_x^2 + I_y^2} \quad (7)$$

$$v^{k+1} = \bar{v}^k - \frac{I_y\left(I_x\bar{u}^k + I_y\bar{v}^k + I_t\right)}{\alpha^2 + I_x^2 + I_y^2} \quad (8)$$

- $\bar{u}^k$, $\bar{v}^k$ are the average velocity values of kth iteration
- Can think of $\frac{dI}{dt}$ as $3 \times N_x \times N_y \times N_t$ continuous tensor field

Bio-inspired (Neumann) model

- $\frac{dI}{dt}$ gradients represented as likelihood values corresponding to membrane potentials of neurons
- Optical flow detection implemented as a three-level process:
  1. Spatiotemporal smoothing/feature extraction via three-stage cascade (V1)
  2. Integration of motion features (asymmetric filter kernels), detection for self-motion, feedback (MT)
  3. Integration of self-motion gradients, feedback (MST)



Bio-inspired (Neumann) model (cont'd)

- We'll focus on first level - three-stage cascade to implement:
- Feedfoward connectivity and temporal filtering

$$\dot{x}^{(1)} = -x^{(1)} + f_{sample}\left([x^{FF}]^{\alpha} * \Lambda_{space} \right) * \Lambda_{vel}. \tag{9}$$

- Feedback connectivity

$$\dot{x}^{(2)} = -x^{(2)} + x^{(1)}(1 + \beta x^{FB}). \tag{10}$$

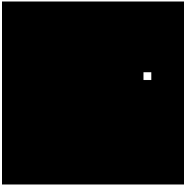
- Lateral (excitatory Λ^+, inhibitory Λ^-) connectivity

$$\dot{x}^{(3)} = -\gamma x^{(3)} + x^{(2)} * \Lambda^+ - x^{(3)}\left(x^{(2)} * \Lambda^-\right), \tag{11}$$

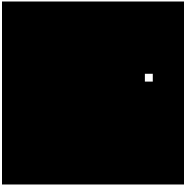
- Can think of likelihood values as $N_x \times N_y \times N_d \times N_v$ continuous tensor field

Gradient detection

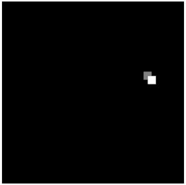
- Use binary image sequence for simplest case of optical flow detection



(a) 1st image



(b) 2nd image

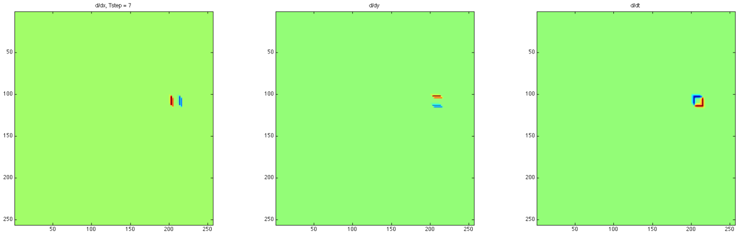


(c) 1st and 2nd

- Use Diderot to store this sequence as a continuous field F and use ∇ operator to verify we can calculate $\frac{dI}{dt}$

Gradient detection (cont'd)

- We implemented a simple function in Diderot to output the gradient of F, and plotted in MATLAB:



- We probed the field at 10 points temporally to get an interpolated flow between 1st and 2nd image

Limitations

- Next step: try iterative updates via Eq. (7) in Horn-Schunk method within Diderot
- Requires local averaging of gradients to yield $[\bar{u}^k, \bar{v}^k]^\top$
- Problem for Diderot - no convenient way to access neighboring points in F and perform average; can't use averaging kernel on a continuous field as it is defined in Diderot
- Even bigger limitation: no shared memory between strands; this prevents inter-neuron connectivity given in Eq. (3) from the neural field formulation, and Eq.(9), (10), and (11)
- Can we use Diderot for something else? $\rightarrow$ Image visualization

Harnessing Diderot

Recall novelties of Diderot:

1. Flexibility of dealing with "continuous" interpretation of discrete data (e.g. limited resolution)
2. Image visualization - high-level math operators common in visualization algorithms are conveniently built in
3. Parallel framework - algorithms that can be run in parallel can be executed quickly

Neuron dynamics and bifurcation

- Phase portrait - way to understand dynamics of conductance-based neuron models

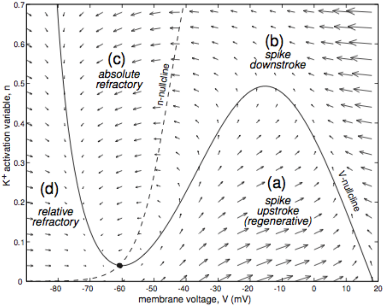


Figure : $I_{Na,p} + I_K$ model, with V and n nullclines [6]

- Compare three models of interest: Hodgkin-Huxley, Morris-Lecar, Fitzhugh-Nagumo

Reduced Hodgkin-Huxley model

Reduced-Hodgkin Huxley model (two-variable V, n):

$$C\dot{V} = I - g_L(V - E_L) - g_{Na}m_\infty(V)^3h_\infty(V)(V - E_{Na}) - g_K n^4(V - E_K)$$
$$\dot{n} = \frac{n_\infty(V) - n}{\tau_n(V)}$$

Morris-Lecar model

Morris-Lecar model:

$$C\dot{V} = I - g_L(V - E_L) - g_{Ca} m_\infty(V)(V - E_{Ca}) - g_K n(V - E_K)$$

$$\dot{n} = \frac{n_\infty(V) - n}{\tau_n(V)}$$

$$m_\infty(V) = \frac{1}{2}\left(1 + \tanh(\frac{V - V_1}{V_2})\right)$$

$$n_\infty(V) = \frac{1}{2}\left(1 + \tanh(\frac{V - V_3}{V_4})\right)$$

$$\tau_n(V) = \phi \cosh\left(\frac{V - V_3}{2V_4}\right)$$

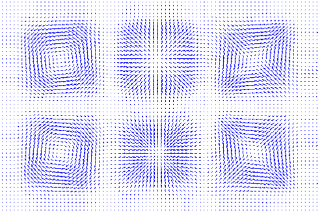
Fitzhugh-Nagumo model

Fitzhugh-Nagumo model:

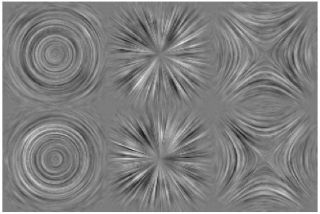
$$\dot{V} = V(a - V)(V - 1) - w + I$$
$$\dot{w} = bV - cw.$$

Line integral convolution

- Gradients figure is convenient for seeing general flow of states, but is distorted to show flow better
- Would be nicer to see this dynamic as a "continuous flow"
- Line integral convolution (LIC) - integrates underlying texture field F along gradient vectors in V to yield more intuitive image



(a) Original vector field



(b) LIC over white noise

Figure : LIC results from Diderot program *lic.diderot*

Line integral convolution (cont'd)

- In general, integrate some kernel function $\Lambda(w)$ over step s^k and step size Δs^k on kth iteration

$$h^k = \int_{s^k}^{s^k+\Delta s^k} \Lambda(w)dw$$

$$P^k = P^{k-1} + \frac{V\left(\lfloor P^{k-1} \rfloor\right)}{\left|\left|V\left(\lfloor P^{k-1} \rfloor\right)\right|\right|} \Delta s^{k-1}$$

$$LIC(x,y) = \frac{\sum_{k=0}^{l_f} F\left(\lfloor P_f^k \rfloor\right) h_f^k + \sum_{k=0}^{l_b} F\left(\lfloor P_b^k \rfloor\right) h_b^k}{\sum_{k=0}^{l_f} h_f^k + \sum_{k=0}^{l_b} h_b^k}$$

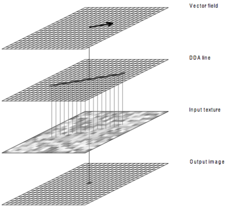
- F - underlying texture field to integrate over, $\lfloor P^k \rfloor$ - floored-position index, V - gradient vector field, h^k - kernel weight, l - number of iterations over k to perform

Line integral convolution (cont'd)

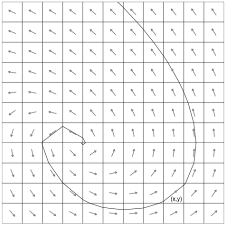
We can simplify the equations by using box kernel and fixed step-size Δs and step number l for forward and backward, with $P_0 = (x, y)$

$$P_f^k = P_f^{k-1} + \Delta s V(P_f^{k-1}), \quad P_b^k = P_b^{k-1} - \Delta s V(P_b^{k-1})$$

$$\Rightarrow LIC(x, y) = \frac{\sum_{k=0}^{2l} F(P_f^k) + F(P_b^k)}{2l + 1}$$



(a) Integration



(b) Stream line [7]

Generating initial phase portraits

- Generate gradient vectors via MATLAB's `quiver`

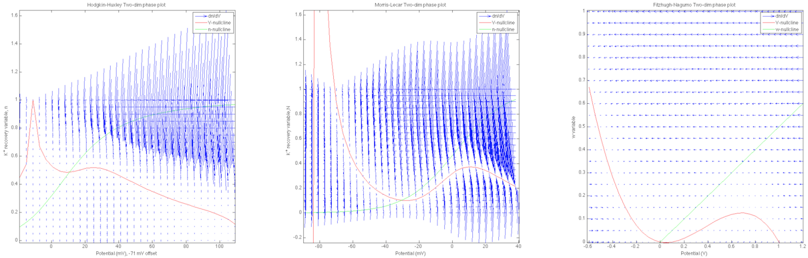


Figure : Un-normalized phase portraits for each neuron model

- Need a normalization to get a better result, without distorting image

Normalization and multi-path limit cycles

- Normalize gradients by a factor of input voltage range

$$dV_{norm} = dV/\eta\big(\max(V) - \min(V)\big). \tag{12}$$

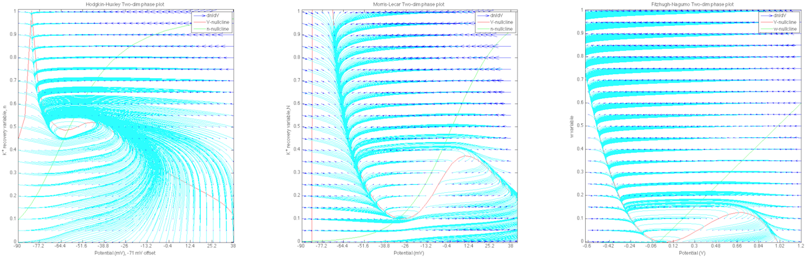
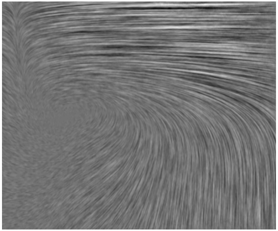


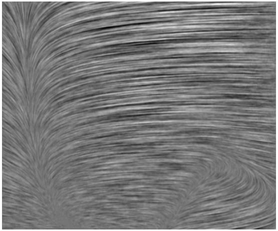
Figure : Normalized phase portraits for each neuron model

- Multi-path limit cycle runs verify the `quiver` vectors

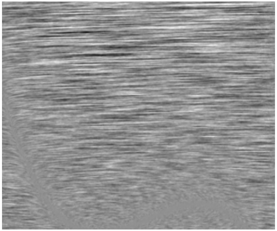
LIC of input phase portraits using Diderot



(a) Hodgkin-Huxley

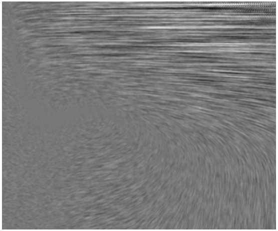


(b) Morris-Lecar

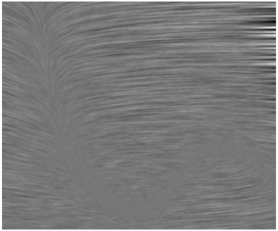


(c) Fitzhugh-Nagumo

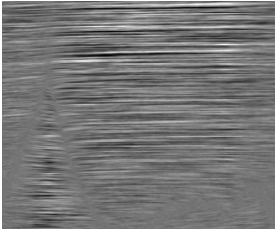
Generating phase portraits/LIC using Diderot (cont'd)



(a) Hodgkin-Huxley



(b) Morris-Lecar



(c) Fitzhugh-Nagumo

Generating phase portraits/LIC using Diderot (cont'd)

- Generating phase portrait/LIC entirely within Diderot was accurate with respect to the plots from MATLAB
- More convenient having results done all in one place
- More accurate using interpolation of $V, \ n$ input range rather than interpolation over vector gradients - functions of nonlinear diff eqs
- Extend this for time-varying injected current - demo videos...

Diderot benchmarking

	Sequential C	Parallel C	OpenCL	CUDA
2.7 GHz Core i5 iMac	30 m 43.8 s	9 m 20 s	Alloc. error	N/A
Huxley Cluster	94 m 39.6 sec	25 m 15 s	Alloc. error	N/A

Table : Benchmark for *licMLInject.diderot*

	Sequential C	Parallel C	OpenCL	CUDA
2.7 GHz Core i5 iMac	7.71 s	2.04 s	59.39 s	N/A
Huxley Cluster	9.59 s	2.71 s	0.85 s	N/A

Table : Benchmark for *licp.diderot*

*Huxley cluster did manage to utilize OpenCL backend to successfully compile and run (reduced size) *licHHInject.diderot*

Summary

- Diderot - great tool for interacting with data from a high-level domain
- Provides flexibility to manipulate data at user-specified granularity, regardless of input resolution
- (Mostly) independent image visualization/analysis algorithms best fit for use in Diderot
- However, not ideal at this time to implement high-level brain operators or perform any sort of sophisticated feature extraction on input sequences
- Despite this limitation, we were able to use Diderot as a visualization tool to bring about a convenient and intuitive way to understand the phase response of various neuron models

Future direction

- Three-dimensional LIC program for phase portrait over the V, n, m plane over Hodgkin-Huxley and Morris-Lecar models, would yield a more sophisticated depiction of the underlying model dynamics.
- LIC is inherently capable of supporting three-dimensional data, though how to visualize the output of the data in a meaningful way? Time-varying input current?
- Diderot's authors: implement inter-strand communication, global mutable memory, CUDA-supported backend, higher memory allocation for OpenCL
- With these functionalities, Diderot would be well equipped for further research in neural encoding and brain operators

Thanks

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