Exploring the Diderot programming language and its applications to the visualization of neural models

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Overview

- Review Diderot - uses, continuous fields, high-level simple syntax, parallelism
- Hypothesis: Can we use Diderot to define high-level brain operators (e.g. optical flow)?
- Neural field equation $\Rightarrow$ Diderot’s continuous tensor field
- Initial experiments and limitations of Diderot
- Visualizing the phase plane with line integral convolution (LIC)
- Experimental results and future direction
Diderot’s intended use

- **Domain Specific Language (DSL)** - designed for image analysis and visualization
- **Image analysis** - Extract *quantitative* geometric properties of objects from image data
- **Image visualization** - use image data in tandem with computer graphics to *qualitatively* describe image properties [1]
- Optimized for algorithms with large number of (mostly) independent computations
- Provides a high-level, simple and direct syntax
Language design and implementation

- Diderot is structured for dealing with data as continuous tensor fields
- Tensor field - scalars (order 0), vectors (order 1), matrices (order 2)
- Formed via convolution of discrete input with "continuous" kernel -
  \[ \text{field} \#k(d)[\sigma] \ F = \text{bspln3} \odot \text{img} \]
- \( k \) - continuous derivatives, \( d \) - dimensionality, \( \sigma \) - tensor order
- Operators on field include -
  \( t = F(pos), \nabla F(pos), \nabla \otimes F(pos); \)
  \( F = \nabla F, \nabla \otimes F, s*F, F_1+F_2; \)
  \( b = \text{inside}(pos,F); \)
- \( t \) is a tensor, which can be manipulated by standard discrete operations, such as - \( \bullet, \otimes, | |, * \)
Actual field computation is performed when field is probed - \( t = \nabla F, \nabla \otimes F, F(pos); \)
Field computation

e.g. \( t = \nabla F(\text{pos}) \); This performs,

\[
\nabla F(x) = (V \otimes \nabla h)(x) \\
= \left( V \otimes \begin{bmatrix} \frac{\partial}{\partial x} h \\ \frac{\partial}{\partial y} h \end{bmatrix} \right) \\
= \left[ \sum_{i=1-s}^{s} \sum_{j=1-s}^{s} V[n+<i,j>]h'(f_{x} - i)h(f_{y} - j) \right] \\
\sum_{i=i-s}^{s} \sum_{j=1-s}^{s} V[n+<i,j>]h(f_{x} - i)h'(f_{y} - j) \right],
\]

where \( V \) - discrete input, \( h \) - (separable) continuous kernel, \( x \) - field position index, \( M^{-1} \) - space mapping matrix, \( n = \lfloor M^{-1}x \rfloor \) discrete mapped point, \( f = M^{-1}x - n \)
Program structure

- 3 sections - global definitions, strand, initialization
- Global - define *immutable* variables, inputs, load image data, usually create field
- Strand - computational core of application - run via parameters, init state variables (including output), update, die, stabilize methods
- A given strand will continue to execute its *update* method until the *stabilize* (write to output) or *die* (no write to output) method is called on that strand
- Initialization - define range of strand parameters (e.g. pixel indices)
Parallelism

- Bulk-synchronous parallelism model - super steps (update method), each consisting of asynchronous computations (individual strands), executes until all strands die or stabilize
- Strands organized into (4096 supported) strands per block, followed by barrier synchronization at end of super step
- Parallel C code, OpenCL, CUDA (future)
Toy program

// ----- Global defs -----
int imgSizeX = 300;    // how many x pts user wants from field
int imgSizeY = 200;    // how many y pts user wants from field
int stepNum = 25;      // step limit before writing to output
image(2)[] img = load("../data/einstein.nrrd"); // import image
field#1(2)[] F = img * ctmr; // convolve img with kernel = field

// ----- Strand section ----- 
strand DEMO (int xi, int yi) {
    real xx = lerp(0.0, 3.0, -0.5, real(xi), real(imgSizeX)-0.5);
    real yy = lerp(0.0, 2.0, -0.5, real(yi), real(imgSizeY)-0.5);
    vec2 pos0 = [xx, yy];
    output real sum = F(pos0);
    int step = 0;
    // --- Update thread with math under conditions ---
    update {
        // Do some fancy math
        vec2 grad = F (pos);  // take gradient of field and probe it
        sum = grad[0]+grad[1];
        step += 1;
        if (step == stepNum) {
            stabilize;  // write "sum" to output
        }
    }
}

// ----- Initialization section ----- 
initially [ DEMO(xi, yi) | yi in 0..(imgSizeY-1), xi in 0..(imgSizeX-1) ];
Hypotheses

1. We can define high-level operators (analogous to $\nabla$) that will perform encoding of sensory data on a neural circuit.

2. We can represent this circuit as a continuous tensor field.

- Can we use Diderot as a convenient medium for accomplishing this?
Continuous neural fields

- Utilize maturely developed area of continuous dynamical systems to approximate neural circuitry.
- Continuous equations - able to model neural activity as quantities in a continuous field, with functional relationships to sources and sinks of that field [3].
- Basic idea: activity of a neuron unit (e.g. average firing rate) in field layer $i$ at point $x$, time $t$ is:

$$Z_i(x, t) = f_i(u_i(x, t))$$

- $u_i(x, t)$ is average membrane potential, $f_i$ is a non-linear activation function [4].
Continuous neural fields (cont’d)

- Model also assumes local excitatory and distant inhibitory inter-connectivity between neurons, described by time-varying weight function
- \( w_{ij}(x, y; t) \) - influence activity from neuron at point \( y \) in layer \( j \) has on neuron at point \( x \) in layer \( i \), \( t \) time units after firing initiates from neuron \( y \)

(a) Neural layers

(b) Weight function
High-level brain operators

Continuous neural fields

Continuous neural fields (cont’d)

- Canonical continuous field equation [4]:

\[
\tau_i \frac{\partial u_i(x, t)}{\partial t} = h_i + \Delta s_i(x, t) - u_i(x, t) + \sum_{j=1}^{m} \int_{\Omega_j} \int_{v} w_{ij}(x, y; t - v) Z_j(y, v) \, dv \, dy
\]  

- \( \tau_i \) is the recovery time to steady-state, \( \Delta s_i(x, t) \) is change in external stimulus, \( h_i \) is average distance to steady-state activity, \( \Omega_j \) spatial neighborhood of summation on layer \( j \)

- Can think of neural field as \( N_x \times N_t \times N_j \) continuous tensor field
Optical flow

- If we can define a neural circuit, such as in Eq. (3), what can we do with it?
- Put operation of continuous field into context - specific objective: encode optical flow
- Optical flow - quantification of relative motion in a vision sequence
In general, three stages in detecting optical flow:

1. Pre-filter/smooth input data spatiotemporally
2. Extract relevant features (e.g. gradients)
3. Weight and integrate features to produce field flow vectors

Known, in general, as a differential or gradient-based approach
Horn-Schunk method

- Gradient constraint: change along the spatiotemporal path of a point of intensity is zero.

\[
\frac{dl(x, t)}{dt} = 0 \Rightarrow \nabla_x l(x, t) \cdot v + \frac{\partial l(x, t)}{\partial t} = 0 \tag{4}
\]

\[
\nabla_x l(x, t) = \begin{bmatrix}
\frac{\partial l(x,t)}{\partial x} \\
\frac{\partial l(x,t)}{\partial y}
\end{bmatrix}, \quad v = \begin{bmatrix} u \\ v \end{bmatrix} \tag{5}
\]

- Horn and Schunk posed this as an optimization problem with smoothness constraint, given by \( \lambda \) [5]

\[
\min_v \int_\Omega (\nabla l \cdot v + \frac{\partial l}{\partial t})^2 + \lambda^2 (\|\nabla u\|_2^2 + \|\nabla v\|_2^2) \, dx \tag{6}
\]
Horn-Schunk method (cont’d)

- HS iterative algorithm:

\[
\begin{align*}
    u^{k+1} &= \bar{u}^k - \frac{l_x (l_x \bar{u}^k + l_y \bar{v}^k + l_t)}{\alpha^2 + l_x^2 + l_y^2} \\
    v^{k+1} &= \bar{v}^k - \frac{l_y (l_x \bar{u}^k + l_y \bar{v}^k + l_t)}{\alpha^2 + l_x^2 + l_y^2}
\end{align*}
\]

(7) (8)

- \(\bar{u}^k, \bar{v}^k\) are the average velocity values of \(k\)th iteration

- Can think of \(\frac{dI}{dt}\) as \(3 \times N_x \times N_y \times N_t\) continuous tensor field
Bio-inspired (Neumann) model

- $\frac{dl}{dt}$ gradients represented as likelihood values corresponding to membrane potentials of neurons

- Optical flow detection implemented as a three-level process:
  1. Spatiotemporal smoothing/feature extraction via three-stage cascade (V1)
  2. Integration of motion features (asymmetric filter kernels), detection for self-motion, feedback (MT)
  3. Integration of self-motion gradients, feedback (MST)
Bio-inspired (Neumann) model (cont’d)

- We’ll focus on first level - three-stage cascade to implement:
  - Feedfoward connectivity and temporal filtering
    \[ \dot{x}^{(1)} = -x^{(1)} + f_{\text{sample}} \left( [x^{FF}]^{\alpha} \ast \Lambda_{\text{space}} \right) \ast \Lambda_{\text{vel}}. \] (9)
  - Feedback connectivity
    \[ \dot{x}^{(2)} = -x^{(2)} + x^{(1)}(1 + \beta x^{FB}). \] (10)
  - Lateral (excitatory \( \Lambda^+ \), inhibitory \( \Lambda^- \)) connectivity
    \[ \dot{x}^{(3)} = -\gamma x^{(3)} + x^{(2)} \ast \Lambda^+ - x^{(3)}(x^{(2)} \ast \Lambda^-), \] (11)

- Can think of likelihood values as \( N_x \times N_y \times N_d \times N_v \) continuous tensor field
Gradient detection

- Use binary image sequence for simplest case of optical flow detection

(a) 1st image  
(b) 2nd image  
(c) 1st and 2nd

- Use Diderot to store this sequence as a continuous field $F$ and use $\nabla$ operator to verify we can calculate $\frac{dl}{dt}$
Gradient detection (cont’d)

- We implemented a simple function in Diderot to output the gradient of $F$, and plotted in MATLAB:

- We probed the field at 10 points temporally to get an interpolated flow between 1st and 2nd image
Limitations

- Next step: try iterative updates via Eq. (7) in Horn-Schunk method within Diderot
- Requires local averaging of gradients to yield $[\bar{u}^k, \bar{v}^k]^\top$
- Problem for Diderot - no convenient way to access neighboring points in $F$ and perform average; can’t use averaging kernel on a continuous field as it is defined in Diderot
- Even bigger limitation: no shared memory between strands; this prevents inter-neuron connectivity given in Eq. (3) from the neural field formulation, and Eq. (9), (10), and (11)
- Can we use Diderot for something else? → Image visualization
Harnessing Diderot

Recall novelties of Diderot:

1. Flexibility of dealing with “continuous” interpretation of discrete data (e.g. limited resolution)
2. Image visualization - high-level math operators common in visualization algorithms are conveniently built in
3. Parallel framework - algorithms that can be run in parallel can be executed quickly
Neuron dynamics and bifurcation

- Phase portrait - way to understand dynamics of conductance-based neuron models

Figure: $I_{Na,p} + I_K$ model, with $V$ and $n$ nullclines [6]

- Compare three models of interest: Hodgkin-Huxley, Morris-Lecar, Fitzhugh-Nagumo
Reduced-Hodgkin Huxley model (two-variable $V, n$):

\[
\begin{align*}
C \frac{dV}{dt} &= I - g_L(V - E_L) - g_{Na} m_\infty(V)^3 h_\infty(V)(V - E_{Na}) - g_K n^4(V - E_K) \\
\dot{n} &= \frac{n_\infty(V) - n}{\tau_n(V)}
\end{align*}
\]
Morris-Lecar model:

\[
\begin{align*}
C \dot{V} &= I - g_L(V - E_L) - g_{Ca} m_\infty(V)(V - E_{Ca}) - g_K n(V - E_K) \\
\dot{n} &= \frac{n_\infty(V) - n}{\tau_n(V)} \\
m_\infty(V) &= \frac{1}{2} \left( 1 + \tanh \left( \frac{V - V_1}{V_2} \right) \right) \\
n_\infty(V) &= \frac{1}{2} \left( 1 + \tanh \left( \frac{V - V_3}{V_4} \right) \right) \\
\tau_n(V) &= \phi \cosh \left( \frac{V - V_3}{2V_4} \right)
\end{align*}
\]
Fitzhugh-Nagumo model:

\[
\begin{align*}
\dot{V} &= V(a - V)(V - 1) - w + I \\
\dot{w} &= bV - cw.
\end{align*}
\]
Line integral convolution

- Gradients figure is convenient for seeing general flow of states, but is distorted to show flow better
- Would be nicer to see this dynamic as a “continuous flow”
- Line integral convolution (LIC) - integrates underlying texture field $F$ along gradient vectors in $V$ to yield more intuitive image

(a) Original vector field
(b) LIC over white noise

Figure: LIC results from Diderot program *lic.diderot*
In general, integrate some kernel function $\Lambda(w)$ over step $s^k$ and step size $\Delta s^k$ on $k$th iteration

$$h^k = \int_{s^k}^{s^k+\Delta s^k} \Lambda(w)dw$$

$$P^k = P^{k-1} + \frac{V(\lfloor P^{k-1} \rfloor)}{\|V(\lfloor P^{k-1} \rfloor)\|} \Delta s^{k-1}$$

$$LIC(x, y) = \frac{\sum_{k=0}^{l_f} F(\lfloor P^k_f \rfloor) h^k_f + \sum_{k=0}^{l_b} F(\lfloor P^k_b \rfloor) h^k_b}{\sum_{k=0}^{l_f} h^k_f + \sum_{k=0}^{l_b} h^k_b}$$

- $F$ - underlying texture field to integrate over,
- $\lfloor P^k \rfloor$ - floored-position index,
- $V$ - gradient vector field,
- $h^k$ - kernel weight,
- $l$ - number of iterations over $k$ to perform.
Line integral convolution (cont’d)

We can simplify the equations by using box kernel and fixed step-size $\Delta s$ and step number $l$ for forward and backward, with $P_0 = (x, y)$

\[
P_f^k = P_f^{k-1} + \Delta s V(P_f^{k-1}), \quad P_b^k = P_b^{k-1} - \Delta s V(P_b^{k-1})
\]

\[
\Rightarrow LIC(x, y) = \frac{\sum_{k=0}^{2l} F(P_f^k) + F(P_b^k)}{2l + 1}
\]

(a) Integration  (b) Stream line [7]
Generating initial phase portraits

- Generate gradient vectors via MATLAB’s `quiver`

**Figure**: Un-normalized phase portraits for each neuron model

- Need a normalization to get a better result, without distorting image
Normalization and multi-path limit cycles

- Normalize gradients by a factor of input voltage range

\[ dV_{\text{norm}} = \frac{dV}{\eta (\max(V) - \min(V))}. \]  \hspace{1cm} (12)

**Figure**: Normalized phase portraits for each neuron model

- Multi-path limit cycle runs verify the *quiver* vectors
LIC of input phase portraits using Diderot

(a) Hodgkin-Huxley  (b) Morris-Lecar  (c) Fitzhugh-Nagumo
Generating phase portraits/LIC using Diderot (cont’d)

(a) Hodgkin-Huxley  
(b) Morris-Lecar  
(c) Fitzhugh-Nagumo
Generating phase portraits/LIC using Diderot (cont’d)

- Generating phase portrait/LIC entirely within Diderot was accurate with respect to the plots from MATLAB
- More convenient having results done all in one place
- More accurate using interpolation of $V$, $n$ input range rather than interpolation over vector gradients - functions of nonlinear diff eqs
- Extend this for time-varying injected current - demo videos...
## Diderot benchmarking

<table>
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<th></th>
<th>Sequential C</th>
<th>Parallel C</th>
<th>OpenCL</th>
<th>CUDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7 GHz Core i5 iMac</td>
<td>30 m 43.8 s</td>
<td>9 m 20 s</td>
<td>Alloc. error</td>
<td>N/A</td>
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<tr>
<td>Huxley Cluster</td>
<td>94 m 39.6 sec</td>
<td>25 m 15 s</td>
<td>Alloc. error</td>
<td>N/A</td>
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</table>

**Table**: Benchmark for `licMLInject.diderot`

<table>
<thead>
<tr>
<th></th>
<th>Sequential C</th>
<th>Parallel C</th>
<th>OpenCL</th>
<th>CUDA</th>
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</thead>
<tbody>
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<td>2.7 GHz Core i5 iMac</td>
<td>7.71 s</td>
<td>2.04 s</td>
<td>59.39 s</td>
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<td>Huxley Cluster</td>
<td>9.59 s</td>
<td>2.71 s</td>
<td>0.85 s</td>
<td>N/A</td>
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</table>

**Table**: Benchmark for `licp.diderot`

*Huxley cluster did manage to utilize OpenCL backend to successfully compile and run (reduced size) `licHHInject.diderot`*
Summary

- Diderot - great tool for interacting with data from a high-level domain
- Provides flexibility to manipulate data at user-specified granularity, regardless of input resolution
- (Mostly) independent image visualization/analysis algorithms best fit for use in Diderot
- However, not ideal at this time to implement high-level brain operators or perform any sort of sophisticated feature extraction on input sequences
- Despite this limitation, we were able to use Diderot as a visualization tool to bring about a convenient and intuitive way to understand the phase response of various neuron models
Future direction

- Three-dimensional LIC program for phase portrait over the $V, n, m$ plane over Hodgkin-Huxley and Morris-Lecar models, would yield a more sophisticated depiction of the underlying model dynamics.
- LIC is inherently capable of supporting three-dimensional data, though how to visualize the output of the data in a meaningful way? Time-varying input current?
- **Diderot’s authors**: implement inter-strand communication, global mutable memory, CUDA-supported backend, higher memory allocation for OpenCL
- With these functionalities, Diderot would be well equipped for further research in neural encoding and brain operators
Thanks

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